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Different Forms of Mathematical Induction

Mathematical induction can be informally illustrated by reference to the sequential effect of falling dominoes.

Mathematical induction is the method of mathematical proof typically used to establish a given statement for all natural numbers. It is done in two steps. The first step, known as the base case, is to prove the given statement for the first natural number. The second step, known as the inductive step, is to prove that the given statement for any one natural number implies the given statement for the next natural number. From these two steps, mathematical induction is the rule from which we infer that the given statement is established for all natural numbers.

The method can be extended to prove statements about more general well-founded structures, such as trees; this generalization, known as structural induction, is used in mathematical logic and computer science. Mathematical induction in this extended sense is closely related to recursion. Mathematical induction, in some form, is the foundation of all correctness proofs for computer programs.

In 370 BC, Plato's Parmenides may have contained an early example of an implicit inductive proof. The earliest implicit traces of mathematical induction can be found in Euclid'sproof that the number of primes is infinite and in Bhaskara's "cyclic method".

An implicit proof by mathematical induction for arithmetic sequences was introduced in the *al-Fakhri* written by al-Karaji around 1000 AD, who used it to prove the binomial theoremand properties of Pascal's triangle.

The simplest and most common form of mathematical induction infers that a statement involving a natural number *n* holds for all values of *n*. The proof consists of two steps:

1. The basis (base case): prove that the statement holds for the first natural number *n*. Usually, *n* = 0 or *n* = 1.
2. The inductive step: prove that, if the statement holds for some natural number *n*, then the statement holds for *n* + 1.

The hypothesis in the inductive step that the statement holds for some *n* is called the induction hypothesis (or inductive hypothesis). To perform the inductive step, one assumes the induction hypothesis and then uses this assumption to prove the statement for *n* + 1.

Whether *n* = 0 or *n* = 1 depends on the definition of the natural numbers. If 0 is considered a natural number, as is common in the fields of combinatorics and mathematical logic, the base case is given by *n* = 0. If, on the other hand, 1 is taken as the first natural number, then the base case is given by *n* = 1.

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