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Extremal problems for partially non-overlapping domains on equiangular system points.

Let $\mathbb{N}$, $\mathbb{R}$ – the sets natural and real numbers conformity, $\mathbb{C}$ – the plain complex numbers, $\overline{\mathbb{C}} = \mathbb{C}\bigcup\{\infty\}$ – the Riemannian sphere, $\mathbb{R}_+ = (0, \infty)$.

For fix number $n \in \mathbb{N}$ system points

$$A_n = \{a_k \in \mathbb{C} : k = \overline{1, n}\},$$

the define n-equiangular system points, if by all $k = \overline{1, n}$ realize relation:

$$\arg a_k = \frac{2\pi}{n}(k-1),\ k = \overline{1, n}. \tag{1}$$

System be considered the angular domains:

$$P_k = \{w \in \mathbb{C} : \frac{2\pi}{n}(k-1) < \arg w < \frac{2\pi}{n}k\}, \quad k = \overline{1, n}.$$

For arbitrary n-equiangular system points "controlling" functional to be considered

$$\mu\left(A_n\right) := \prod_{k=1}^{n} \chi\left(\left|\frac{a_{k+1}}{a_k}\right|^{\frac{n}{4}}\right) \cdot |a_k|,$$

where $\chi(t) = \frac{1}{2}(t + \frac{1}{t})$, $t \in \mathbb{R}_+$.

Let D, $D \subset \overline{\mathbb{C}}$ – the arbitrary open set and $w = a \in D$, this $D(a)$ the define connected component D, the contain point a. For arbitrary n-equiangular system points $A_n = \{a_k\}_{k=1}^{n}$ and open set D, $A_n \subset D$ the define $D_k(a_p)$ connected component set $D(a_p)\bigcap\overline{P_k}$, the contain point a_p, $k = \overline{1, n}$, $p = k, k+1$, $s = \overline{1, m}$, $a_{n+1} := a_1$. Let $D_k(0)$ (conformity $D_k(\infty)$) the define connected component set $D(0)\bigcap\overline{P_k}$ (conformity $D(\infty)\bigcap\overline{P_k}$), the contain point $w = 0$ (conformity $w = \infty$).

The define, what open set D, $\{0, \infty\} \cup A_n \subset D$ satisfy the conditions non-overlapping relatively n-equiangular system points A_n if be satisfied condition

$$\left[D_k(a_k)\bigcap D_k(a_{k+1})\right]\bigcup\left[D_k(0)\bigcap D_k(a_k)\right]\bigcup\left[D_k(0)\bigcap D_k(\infty)\right]\bigcup$$
$$\bigcup\left[D_k(\infty)\bigcap D_k(a_k)\right]\bigcup\left[D_k(\infty)\bigcap D_k(a_{k+1})\right]\bigcup\left[D_k(0)\bigcap D_k(a_{k+1})\right] = \varnothing, \tag{2}$$

$k = \overline{1,n}$ on all angular domains $\overline{P_k}$.

System domains $\{B_k\}_{k=1}^{n}$, $k = \overline{1,n}$, the define system partially non-overlapping domains, if

$$D := \bigcup_{k=1}^{n} B_k, \tag{3}$$

is open sets, the satisfied condition (2).

Let $r(B;a)$ – inner radius domain $B \subset \overline{\mathbb{C}}$ with respect to a point $a \in B$.

Theorem. *Let $\gamma \in \mathbb{R}_+$, $n \in \mathbb{N}$, $n \geq 3$. Then for arbitrary n-equiangular system points (1), the satisfied condition*

$$\mu\left(A_n\right) = 1,$$

and arbitrary set partially non-overlapping domains $\{B_0, B_k, B_\infty\}$, the satisfied condition (3), $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{1,n}$, $0 \in B_0 \subset \overline{\mathbb{C}}$, $\infty \in B_\infty \subset \overline{\mathbb{C}}$, be satisfied inequality

$$\left(r\left(B_0\,;\,0\right) \cdot r\left(B_\infty\,;\,\infty\right)\right)^\gamma \cdot \prod_{k=1}^{n} r\left(B_k\,;\,a_k\right) \leq$$

$$\leq \left(r\left(B_0^0\,;\,0\right) \cdot r\left(B_\infty^0\,;\,\infty\right)\right)^\gamma \cdot \prod_{k=1}^{n} r\left(B_k^0\,;\,a_k^0\right).$$

The equality obtain in this inequality, when points $\{a_k^0\}$ and domains $\{B_0^0, B_k^0, B_\infty^0\}$, $k = \overline{1,n}$ are, conformity, the poles and the circular domains of the quadratic differential

$$Q(w)dw^2 = -\frac{\gamma w^{2n} + \left(n^2 - 2\gamma\right) w^n + \gamma}{w^2\left(w^n - 1\right)^2} dw^2.$$