MINISTRY OF EDUCATION, SCIENCE, YOUTH
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DONETSK NATIONAL UNIVERSITY

INTERNATIONAL CONFERENCE IN MODERN ANALYSIS

Abstracts

June 20-23, 2011

Donetsk, Ukraine

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The Väisälä inequality for mappings with finite length distortion

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The present talk is devoted to the study of space mappings $f(x) = (f_1(x), \dots, f_n(x))$ defined in a domain $D \subset \mathbb{R}^n$, $n \geq 2$, i.e., $x = (x_1, \dots, x_n) \in D$. In what follows, D be a domain in $\mathbb{R}^n$, $n \geq 2$, and m be a measure of Lebesgue in $\mathbb{R}^n$. A mapping $f : D \to \mathbb{R}^n$ is said to be *discrete* if the preimage $f^{-1}(y)$ of every point $y \in \mathbb{R}^n$ consists of isolated points, and an *open* if the image of every open set $U \subset D$ is open in $\mathbb{R}^n$. We suppose that $f : D \to \mathbb{R}^n$ is continuous.

Recall that a mapping $f : D \to \mathbb{R}^n$ is said to have the N – *property (of Luzin)* if $m(f(S)) = 0$ whenever $m(S) = 0$ for all such sets $S \subset \mathbb{R}^n$. Similarly, f has the N^{-1} – *property* if $m(S) = 0$ whenever $m(f(S)) = 0$. A mapping $f : D \to \mathbb{R}^n$ is said to be of *finite metric distortion*, abbr. $f \in FMD$, if f is differentiable a.e. and has N – and N^{-1} – property.

A path γ in $\mathbb{R}^n$ is a continuous mapping $\gamma : \Delta \to \mathbb{R}^n$ where Δ is an interval in $\mathbb{R}$. Its locus $\gamma(\Delta)$ is denoted by $|\gamma|$. Given a family of paths Γ in $\mathbb{R}^n$, a Borel function $\rho : \mathbb{R}^n \to [0, \infty]$ is called *admissible* for Γ, abbr. $\rho \in \operatorname{adm}\Gamma$, if curvilinear integral of the first type $\int\limits_{\gamma} \rho(x)|dx| \geq 1$ for each $\gamma \in \Gamma$. The *modulus* $M(\Gamma)$ of Γ is defined as $M(\Gamma) = \inf\limits_{\rho \in \operatorname{adm}\Gamma} \int\limits_{\mathbb{R}^n} \rho^n(x)\,dm(x)$ interpreted as $+\infty$ if $\operatorname{adm}\Gamma = \varnothing$. We say that a property P holds for *almost every (a.e.)* path γ in a family Γ if the subfamily of all paths in Γ for which P fails has modulus zero.

If $\gamma : \Delta \to \mathbb{R}^n$ is a locally rectifiable path, then there is the unique increasing length function l_γ of Δ onto a length interval $\Delta_\gamma \subset \mathbb{R}$ with a prescribed normalization $l_\gamma(t_0) = 0 \in \Delta_\gamma$, $t_0 \in \Delta$, such that $l_\gamma(t)$ is equal to the length of the subpath $\gamma|_{[t_0,t]}$ of γ if $t > t_0$, $t \in \Delta$, and $l_\gamma(t)$ is equal to $-l(\gamma|_{[t,t_0]})$ if $t < t_0$, $t \in \Delta$. Let $g : |\gamma| \to \mathbb{R}^n$ be a continuous mapping, and suppose that the path $\widetilde{\gamma} = g \circ \gamma$ is also locally rectifiable. Then there is a unique increasing function $L_{\gamma,g} : \Delta_\gamma \to \Delta_{\widetilde{\gamma}}$ such that $L_{\gamma,g}(l_\gamma(t)) = l_{\widetilde{\gamma}}(t) \quad \forall \quad t \in \Delta$. A path γ in D is called here a *lifting* of a path $\widetilde{\gamma}$ in $\mathbb{R}^n$ under $f : D \to \mathbb{R}^n$ if $\widetilde{\gamma} = f \circ \gamma$. Recall that $f \in ACP$ if and only if $L_{\gamma,f}$ is absolutely continuous on closed subintervals of Δ_γ for a.e. path γ in D. We say that f is *absolute continuous on paths in the inverse direction,* abbr. ACP^{-1}, if $L^{-1}_{\gamma,f}$ is absolutely continuous on closed subintervals of $\Delta_{\widetilde{\gamma}}$ for a.e. path $\widetilde{\gamma}$ in $f(D)$ and for each lifting γ of $\tilde{\gamma}$. It is said that a discrete mapping $f : D \to \mathbb{R}^n$ has the (L) – *property* if $f \in ACP \cap ACP^{-1}$. A mapping $f : D \to \mathbb{R}^n$ is said to be of *finite length distortion*, abbr. $f \in FLD$, if f is FMD and has the (L) – property.

In what follows $f'(x)$ denotes the Jacobian matrix of f, $J(x, f)$ is its determinant and $l(f'(x)) = \min\{|f'(x)h| : h \in \mathbb{R}^n, |h| = 1\}$. The *inner dilatation* of f at the point x is defined as $K_I(x, f) = \frac{|J(x,f)|}{l(f'(x))^n}$, if $J(x, f) \neq 0$, $K_I(x, f) = 1$, if $f'(x) = 0$ and $K_I(x, f) = \infty$ at the rest points. A domain $G \subset D$, such that $\overline{G} \subset D$, is said to be a *normal domain of* f, if $\partial f(G) = f(\partial G)$. Set $N(y, f, E) = \operatorname{card}\{x \in E : f(x) = y\}$, $N(f, E) = \sup\limits_{y \in \mathbb{R}^n} N(y, f, E)$.

Theorem. *Let $f : D \to \mathbb{R}^n$ be a discrete open mapping of finite length distortion, $G \subset D$ is a normal domain for f, Γ' be a path family in $G' = f(G)$, Γ be a path family α in G such that $f \circ \alpha \subset \Gamma'$. Then*

$$M(\Gamma') \quad \leq \quad \frac{1}{N(f, G)} \quad \int_G K_I(x,\, f) \cdot \rho^n(x) \quad dm(x)$$

for every $\rho \in \operatorname{adm}\Gamma$.