I. I. Polischuk
research supervisor: T.V Didkivska, associate professor, candidate of physical-mathematical sciences
I. Franko Zhytomyr State University
Language tutor: L.P. Serdiichuk

DIOPHANTINE EQUATIONS

The problem of Diophantine equations was studied by a number of outstanding matematicians: Pythagoras, Diophantus, L. Euler (1601 – 1665), Fermat (1707 – 1783), J. L. Lagrange (1736 – 1813), K. F. Gauss (1777 – 1855), P. L. Chebyshev (1821 – 1894) and others. There is no general method of finding solutions of Diophantine equations till now [2].

There are a lot of ways of treating of the Diophantine equations and methods of their solutions.

The purpose of any equation is to solve all the unknowns in the problem or find that there is no solution. A Diophantine problem is one in which the solutions are required to be integers. As the name suggests, many problems that we now call Diophantine equations are addressed in the Arithmetica of Diophantus. However, some of these problems were known well before the time of Diophantus. Also, some of the most famous problems of number theory, such as Fermat's Last Theorem, are Diophantine equations posed by mathematicians living much later.

Definition. Let $P(x, y, ...)$ is a polynomial with integer coefficients in one or more variables. A Diophantine equation is an algebraic equation $P(x, y, z, ...) = 0$ for which integer solutions are sought.

For example, $3x + 11y = 53$; $13x^2 - 7y^2 + 3x + 5y - 2 = 0$; $y^3 + x^3 = z^3$

Linear Diophantine Equations Definition. A linear Diophantine equation (in two variables x and y) is an equation $ax + by = c$ with integer coefficients $a, b, c \in \mathbb{Z}$ to which we seek integer solutions [2].

Brahmagupta, the Indian mathematician, was the first who gave the general solution of the linear Diophantine equation $ax + by = c$.

It is not obvious that all such equations are solvable. For example, the equation $2x + 2y = 1$ does not have any integer solutions. Some linear Diophantine equations have finite number of solutions, for example $2x = 4$ and some have infinite number of solutions.

Theorem. Let $a, b, c \in \mathbb{Z}$. Consider the Diophantine equation $ax + by = c$

(a) If $(a, b) \nmid c$, there are no solutions.

(b) If $(a, b) = d \mid c$ (that means c is divided entirely on GCD$(a, b) = d$ – the greatest common division GCD(a, b)), there are infinitely many solutions of the form

$$x = x_0 + \frac{b}{d}k,\ y = y_0 - \frac{a}{d}k.$$ Here (x_0, y_0) is a particular solution, and $k \in \mathbb{Z}$ [1].

The study of the material showed that we can find all integer solutions of Diophantine equations by a number of methods: the Extended Euclidean Algorithm, a scattering method, the use of continued fractions, a method of dividing on the smallest coefficient, a remainder method, a comparative method.

So, Diophantine equations can be used to find all possible solutions of many problems like those of speed, teamwork, sales, square proportions and so on. In addition, the use of these equations in physics, chemistry and economy is of great importance [2].

LITERATURE

1. Гельфонд А. О. Решение уравнений в целых числах / Александр Йосипович Гельфонд. – М.: Наука, 1978. – 63 с.
2. Серпинский В. О решении уравнений в целых числах / Вацлав Серпинский. – М.: Государственное издательство физико-математической литературы, 1961. – 87 с.