

PERFECT NUMBERS

In ancient times people attached a special mystical significance to the perfect numbers. Due to the complexity of their finding in ancient times they were considered to be divine. Thus, the medieval church believed that the study of perfect numbers led to salvation and those who found the following perfect number would get an eternal bliss. There was also a belief that the world was perfect, because it was created within 6 days. Humanity was though not to be perfect, because it came from the incomplete number 8 as 8 people were saved in Noah's ark. But at the same time 7 pairs of unclean and clean animals were saved and a total of 28 is a perfect number. There are a lot of such matches. For example, a human hand can be called a perfect tool because 10 fingers are 28 phalanges.

The old problem of perfect numbers was mentioned in the "*Introduction to arithmetic*" by Nikomaha.

A *perfect number* is an integer that is equal to the sum of its proper divisors excluding the number itself.

First Pythagoreans knew only two perfect numbers: 6 and 28. It was till the time when Euclid (III in. S. E.) began to study this aspect.

Later Euclid found the following two perfect numbers as 496 and 8128. He also found a general formula for the perfect pair of numbers: (*), which gave the desired result, providing that the number of $2^p - 1$ is simple. Euclid's proof was based on the formula for calculating the sum of geometric progression members^[1].

The perfect numbers 6, 28, 496 and 8128 are derived from Euclid's formula (*) for values of " p " that is equal under 2, 3, 5 and 7:

$$\begin{aligned}2^1 * 2^2 - 1 &= 2 * 3 = 6 \\2^2 * 2^3 - 1 &= 4 * 7 = 28 \\2^4 * 2^5 - 1 &= 16 * 31 = 496 \\2^6 * 2^7 - 1 &= 64 * 127 = 8128\end{aligned}$$

Euclid's formula (*) gives the perfect number not for all simple values. For example, when $p = 11$, number $2^p - 1 = 2^{11} - 1 = 2047 = 23 * 89$ is drawn, therefore, the number $2^{11-1}(2^{11} - 1)$ is not perfect.

The fifth number was found only in 15th century. It was 33 550 336. In Euclid's formula it corresponds to the value $p = 13$.

Mersenne expresses the hypothesis that the next six perfect numbers are derived from the Euclid's formula for values of p which are equal to 17, 19, 31, 67, 127 and 257. For the proof it was necessary to show that at these values, the number $M_p = 2^p - 1$ is simple.

Due to Mersenne's achievements M_p were named after him. In some centuries a Swiss Leonhard Euler managed to check Mersenne's hypothesis for the first three numbers [2].

So far no one has found an odd perfect number and has not proved the existence of it.

Mersenne's hypothesis was wrong. The next Mersenne's number M_{67} , which was not checked by Euler, was drawn up.

In New York, 1903 a session of the American Mathematical Society was held. Professor Frank Cole (1861 - 1926) spoke at this conference. He came to the board and, without saying a word, began raising 2 to power 67. Then he subtracted 1 and moved to a clean part of the board where he multiplied two numbers: $1.937.077.221 * 761.838.857.287$. Both results matched. It was the first and the only time in the history of the society when the audience applauded. Frank Cole without saying a word, took his place. No one asked him any questions. American Mathematical Society established the Cole Prize for outstanding achievements in the number theory.

In 1952 scientists used electronic computers to find Mersenne's numbers. The principle of programming is extremely simple. It is enumerative technique. Five giant Mersenne's primes $M_{521}, M_{607}, M_{1279}, M_{2203}, M_{2281}$ were found with the help of this method.[1]

In conclusion, the history of perfect numbers is pretty intense and rich. There are a lot of unsolved facts and as scientists say there is no enough eternity to check all simple numbers.

LITERATURE

1. Тадеєв В.О. Неформальна математика. 6 – 9 класи. Навчальний посібник для учнів, які хочуть знати більше, ніж вивчається у школі. – Тернопіль: Навчальна книга – Богдан, 2003. – 288 с.
2. У світі математики, Т. 3, в. 1, 1997