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THE EQUATION OF LINEAR DISSIPATIVE OSCILLATOR AND ITS SOLUTION

The topicality of the paper in the fact that the task of the linear oscillator is one of the most important in physics. This problem is studied in the course of theoretical mechanics. This branch of mechanics is mostly often associated with issues of solid state physics, since many tasks of dynamics of elastic systems, crystal lattice and optical media are reduced to the study of simple oscillatory models.

The purpose of the paper is to offer solving of equation of linear dissipative oscillator.

Before solving the equation, let’s consider the definition of a linear oscillator.

A system that implements one-dimensional motion under quasi-elastic force, is called a **linear oscillator**. Examples of linear oscillator are a pendulum, a spring pendulum with damping.

Let’s consider a second example in more details [1].

The friction force for the case of nonlinear dry friction depends on the velocity (Fig.1. a), that’s it’s very difficult to analyze such a system. Let’s introduce a special element with a viscous friction – damper (Fig.2). In the case of damper’s small velocity the dependence of the frictional force on velocity can be considered linear (Fig.1. b). Then the equation of motion for a spring pendulum with damper can be represented according to the second Newton's law

$mx = -kx - hx$ or $x + 2\delta x + \omega_0^2 x = 0, \delta = h \;\; 2m, \omega_0^2 = k \;\; m.$ (1)

This is **the equation of a linear dissipative oscillator**. It can describe, for example, fluctuations of current in the circuit composed of inductance, capacitance and resistance. Let's find the solution of this equation [2].

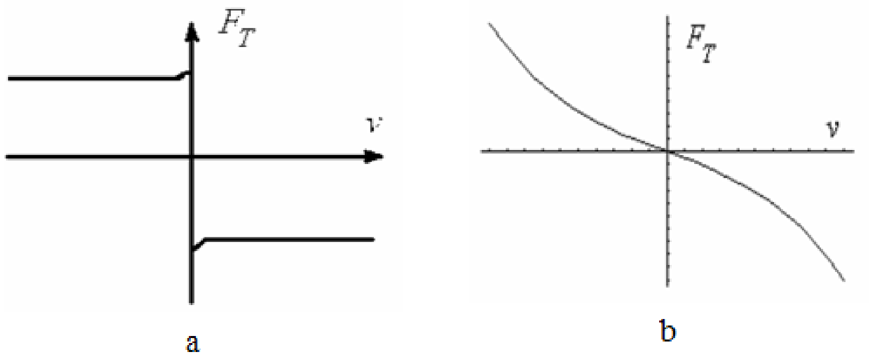

Fig.1. The dependence of the friction force on the speed:
a – the dry friction; b – the viscous friction.

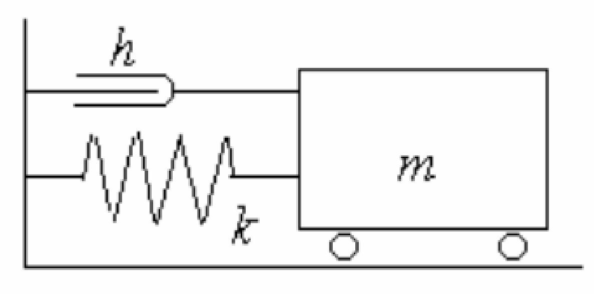

Fig. 2. Spring pendulum with damper.

Equation (1) is can be rewritten easily as a system of two differential equations of the first order (in phase variables):

$$x = y; \qquad y = -2\delta y - \omega_0^2 x. \tag{2}$$

Comparing equation (1) with dynamical systems that are described by the following set of equations

$$x_i = f_i(x_1, \dots, x_n), i = 1,2, \dots, n$$

we see that equation (1) describes a system with one degree of freedom.

The General solution of equation (1) is well known:

$$x(t) = \begin{cases} \left(A\exp(i\omega t) + B\exp(-i\omega t)\right)\exp(-\delta t), \omega = \sqrt{\omega_0^2 - \delta^2}, \omega_0 > \delta; \\ \left(A'\exp(Dt) + B'\exp(-Dt)\right)\exp(-\delta t), D = \sqrt{\delta^2 - \omega_0^2}, \omega_0 < \delta. \end{cases} \tag{3}$$

Arbitrary constants A, B (or A', B') are determined from the initial conditions. In particular, for initial conditions

$$x(t = 0) = X_m, x(t = 0) = 0,$$

we get the following result:

$$A = \left(\frac{1}{2} - i\frac{\delta}{2\omega}\right)X_m;\ B = \left(\frac{1}{2} + i\frac{\delta}{2\omega}\right)X_m;$$

$$A' = \left(\frac{1}{2} + i\frac{\delta}{2D}\right)X_m;\ B' = \left(\frac{1}{2} - i\frac{\delta}{2D}\right)X_m.$$

Dependences (3) for this case are shown in Fig. 3.

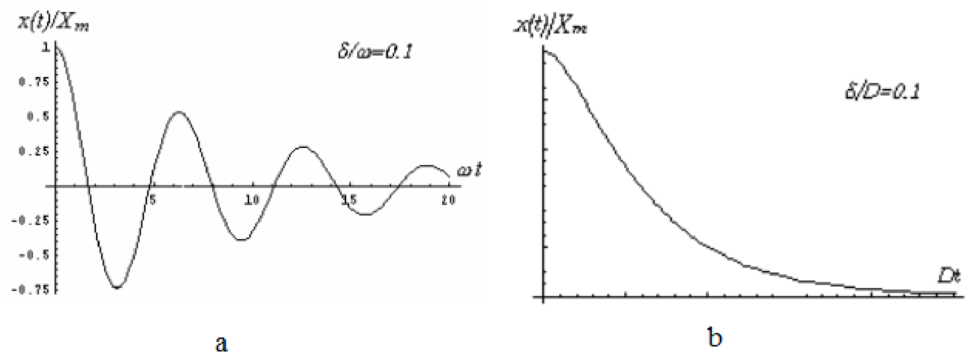

Fig. 3. Free vibrations of a linear dissipative oscillator for cases $\omega_0 > \delta$ (a) and $\omega_0 < \delta$ (b).

Resume. This paper presents the definition of the linear oscillator and its examples. The solution of linear dissipative oscillator's equation is found. The perspectives of the further work are as follows:

- the derivation of the equation of a conservative oscillator on condition of free oscillations, and

- the studying of the forced oscillations of the linear oscillator.

LITERATURE

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