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THE USAGE OF A VECTOR METHOD FOR SOLVING PLANIMETRIC PROBLEMS

A vector is a very important part of school curriculum in Mathematics. There are many definitions of vectors. That is why it is necessary to choose such a definition of a vector that will be appropriate to the topic of our research. As our article is connected with planimetrics, we will use a definition of a vector on plane. According to this definition, a vector is a line segment which has a determined direction [1].

It is very important to know the operations that can be done with vectors. So, we will use such operations as addition and subtraction of vectors, multiplication of vectors by scalar and scalar multiplication of vectors.

A vector is the basis of the vector method of solving different mathematical problems. A lot of geometrical problems can be solved with the help of this method for a very short period of time. Besides, the vector method is very interesting. Our article will be focused on some ways of using this method for solving planimetric problems and comparing it with the traditional methods that are studied at school.

The algorithm of solving planimetric problems with the help of the vector method is the following: 1) convert the task into the language of vectors; 2) do the operations with vectors and use the well-known inequalities with vectors; 3) convert the result of the previous operations from the language of vectors into the language of traditional geometry [3].

All problems of planimetrics can be divided into two groups: the problems that are formulated in the language of geometry and the problems that are formulated in the language of vectors (in this case we should start solving the problem from item #2 in the algorithm given above). Let's solve one of the planimetric problems with the help of the vector method and traditional methods of geometry.

Problem: We have a triangle with sides which are equal to a, b, c. We should find the medians m_a , m_b , m_c that are drawn to these sides [2].

Method #1 (a traditional method of geometry):

$$a^{2} = \frac{b^{2}}{4} + m_{b}^{2} - bm_{b}\cos(180 - \alpha) = \frac{b^{2}}{4} + m_{b}^{2} - bm_{b}(-\cos(-\alpha)) = \frac{b^{2}}{4} + m_{b}^{2} + bm_{b}\cos(-\alpha) = \frac{b^{2}}{4} + m_{b}^{2} + bm_{b}\cos(\alpha).$$

$$c^{2} + a^{2} = \frac{b^{2}}{2} + 2m_{b}^{2}$$

$$m_{b} = \frac{1}{2}\sqrt{2a^{2} + 2c^{2} - b^{2}}$$

$$m_{a} = \frac{1}{2}\sqrt{2b^{2} + 2c^{2} - a^{2}}, m_{c} = \frac{1}{2}\sqrt{2a^{2} + 2b^{2} - c^{2}}$$

Method #2 (a vector method):

Let us introduce the $\overrightarrow{BA} = \overrightarrow{c}$, $\overrightarrow{BC} = \overrightarrow{a}$, $\overrightarrow{AC} = \overrightarrow{b}$, $\overrightarrow{BD} = \overrightarrow{m_b}$ vectors:

$$\begin{cases}
\vec{a} + \vec{c} = 2\vec{m}_b, \\
\vec{a} - \vec{c} = \vec{b}.
\end{cases} \implies \begin{cases}
a^2 + 2\vec{a}\vec{c} + c^2 = 4m_b^2, \\
a^2 - 2\vec{a}\vec{c} + c^2 = b^2.
\end{cases}$$

$$2a^2 + 2c^2 = 4m_b^2 + b^2, \\
m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}, m_c = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2}$$

There are a lot of geometrical problems that can be solved using the vector method but it doesn't mean that such a method is optimal for all problems. In some cases it can only make solving the problem more complicated. Therefore, we should analyze cons and pros of using the vector method in all particular cases.

LITERATURE

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