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GRAPH THEORY IN LOGICAL PROBLEMS

The first work on the graph theory belongs to Leonard Euler and it was published in 1736. Firstly, graph theory was used only for certain logical tasks and puzzles. But already in the 19th century it became clear that graph theory could be used to construct molecular and electrical circuits. Wide application of the graph theory has also been found in programming (any block diagram can be specified by the graph) and in genetics (for the decoding of DNA) and in other fields.

We will consider several problems that are convenient to solve with the help of graph method. To begin with, let's remember what a graph is. If there are several points and lines on the plane and each of them connects a pair of given points, then we could say that the graph is specified. A graph is a finite set of points and arcs that connects these points. Each vertex has its own index and it is determined by the number of edges that converge to this vertex.

Let's consider **the problem of Euler** (1707 – 1783).

The city of Königsberg is located on the banks of the Pregel River and on two islands. Different parts of the city connect 7 bridges. Every Sunday residents of the city walk around the city. Is it possible to devise a way in which they could walk around the city, crossing each of the 7 bridges only once? [2]

A graph that can be passed by a continuous motion, without passing the same edge twice is called unicursal. Euler proved that if the graph is

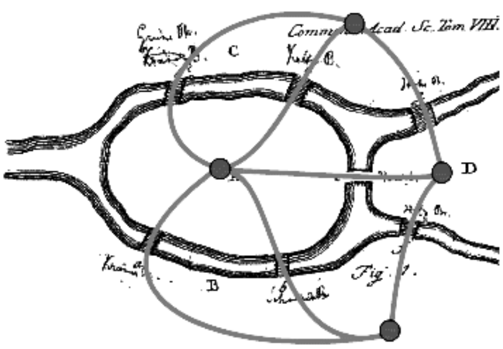

Figure 1

unicursal, it should have no more than 2 vertices of the odd index. So, it can be seen from **Figure 1** that the desired route does not exist, because all 4 vertices have an odd index.

Let's consider **the Poisson problem**. It is believed that, after being interested in this task, Siméon Poisson (1781-1810) began to work in mathematics and devoted all his life to it.

Someone has 12 pints of wine and wants to gift half of that amount, but he does not have a jug of 6 pints. He has two jugs: one for 8, the other for 5 pints. How to pour 6 pints of wine into a jug of 8 pints? What is the smallest number of transitions needed to be done?

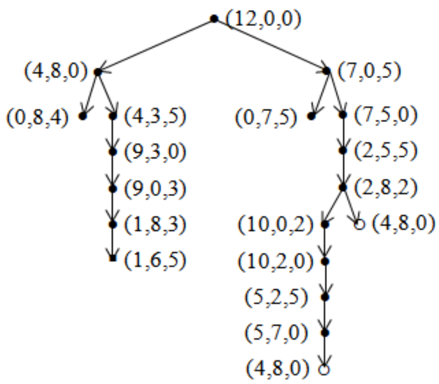

Figure 2

In **Figure 2**, you can see all the possible options for filling wine jugs. A triple of numbers denotes three jugs. The first number indicates the number of pints in a jug that holds 12 pints, the second number is the number of pints in a jug that holds 8 pints, and the third number is the number of pints in a jug that holds 5 pints.

The figure shows that only one variant of such a transfusion is possible (the second option is reduced to it, but the number of transfusions will be much larger). And the smallest number of transfusions that you need to do consists of 6 shifts.

That is, you need to do the following:

- pour wine from a 12-pint jug to an 8-pint (4,8,0);
- from 8-pint to 5-pint (4,3,5);
- from the 5-pint back to the 12-pint (9,3,0);
- pour 3 pints from 8 pints to 5 pints (9,0,3);
- pour wine from a 12-pint to an 8-pint jug (1,8,3);
- pour a portion of the wine into a 5-pint jug (1,6,5).

There are many similar tasks. All of them can be generalized as follows: let's have two empty jugs with a volume of *a* and *b* liters and you need to dial from the river exactly *c* liters of water. If the number *c* is not divisible by the largest common divisor of the numbers *a* and *b*, then this cannot be done. If *c* is divided into the largest common divisor of the numbers *a* and *b*, then in this case the problem always has a solution. Similarly, the solution will exist if the numbers *a* and *b* are relatively prime.

We see that in the Poisson problem we need to use empty jugs of 8 and 5 pints to draw 6 pints of wine. We know that 8 and 5 are relatively prime numbers, then we see that the problem has a solution. In order to determine if there is a solution to the problem, it does not matter how much wine was originally.

The next known task which can be solved with the help of the graph theory tells us **about a wolf, a goat and a bag of cabbage**.

Carrier (C) must transfer through the river a wolf (W), a goat (G) and a bag of cabbage (B). But the boat is so small that the carrier can take with himself only one of the objects. In addition, the cabbage cannot be left with the goat, and the goat with the wolf. How to make a ferry? [0]

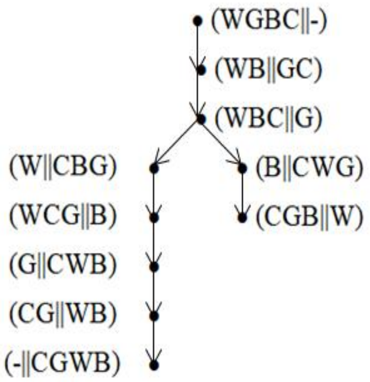

Figure 3

For this task, we will also construct the scheme (**Figure 3**). We see that the problem has two solutions, in particular, the carrier is enough to perform the following steps: first take the goat to another bank, then return and pick up either a bag or the wolf. If the carrier will be near the bank of the river, then the goat will not eat cabbage, and the wolf will not eat a goat. Next, the carrier takes the goat and takes it to the opposite bank, where either the wolf or the bag is taken, respectively. Then he has to pick a goat and then all the objects will be transmitted through the river.

The method of graphs can solve various problems in logic. Let's consider the following:

Three daughters of Yaroslav the Wise - Anne, Anastasia and Elizabeth (Elisiv) - became queens of different countries: Norway, Hungary and France. One of them became the wife of King Harald Sigurdsson, the second - King Henry I, and the third - King Andrew I. Anne's husband was not Harald, and he was not the king of

Hungary. The wife of Harald was not Anastasia. Henry was not married to the Queen of Hungary. The task is to recover spouses and countries if in Scandinavian sources you do not find the names Anastasia and Anna.

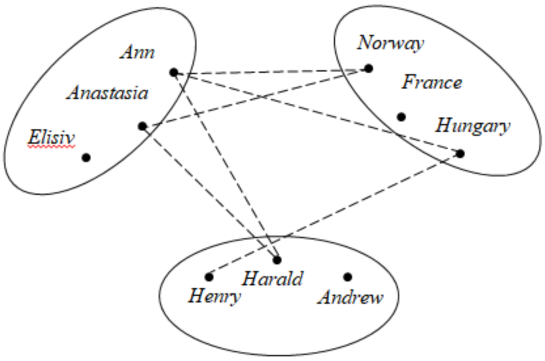

Figure 4

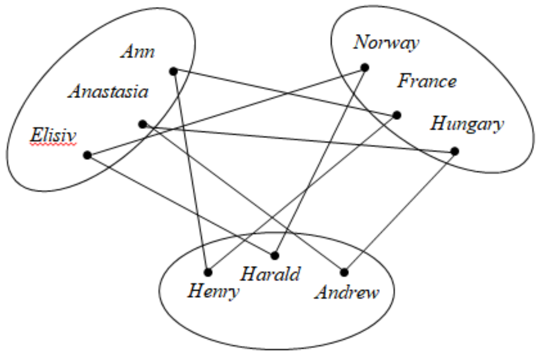

Figure 5

In **Figures 4 and 5**, the vertices of the graph are indicated by the dots, in this case there are 9. The dotted edges denote those vertices that are not in a certain relation, and the edges denoted by the usual line represent the relation between the vertices.

Figure 4 shows all the conditions specified in the task. Then we argue this way: the Queen of Norway is neither Anna nor Anastasia, so Queen of Norway is Elizabeth, and her husband is Harald. Then Henry is Anna's husband and King of France. And, of course, Anastasia's husband is Andrew, and Anastasia is the Queen of Hungary. The answer to the task can be seen in **Figure 5**.

We reviewed the use of graphs to solve some logical problems. This approach can be used to solve problems in mathematics of increased complexity with students of the primary school.

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