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FROM PYTHAGORAS THEOREM TO THE FERMAT THEOREM AND ITS MODIFICATION

In our time, it's hard to find a person who would not have heard of the Pythagorean theorem. It indicates that the sum of the squares of the cateches is equal to the square of the hypotenuse. For the majority of people it will be a great surprise to learn that there is no evidence of the Pythagoras proof of this theorem. But why then it is called so? Pythagoras spent his young years in Egypt for studying. It was there where he learnt about Egyptian triangles ($3^2 + 4^2 = 5^2$). After returning to Greece, he systematized knowledge on this issue and made it's logical justification. Then appeared the Pythagorean trios – the three numbers that satisfy the equality $x^2+y^2=z^2$, for example: (3; 4; 5); (5; 12; 13), (9; 40; 41) or in general:

$$(a^2 - b^2)^2 + (2ab)^2 = (a^2 + b^2)^2$$

If substitute any *a* and *b*, we get an arbitrary Pythagorean trios.

Many years later Pierre Fermat came across this theorem in the second book of "Arithmetic". Being interested in this theorem, he decided to consider other equations of this type: $x^3+y^3=z^3$; $x^4+y^4=z^4$; …; $x^n+y^n=z^n$. Having done this he came to the conclusion, which is now interpreted as the Last Fermat's Theorem [1]:

"The equation $x^n+y^n=z^n$ can not be solved in integers (or rational numbers), provided that the natural $n > 2$ and $xyz \neq 0$".

Unfortunately, he did not leave proofs of his statements, but only short notes. After Fermat's death due to the efforts of his son, his works were published. Realizing all the genius of Fermat, mathematicians all over the world began to prove his statement. From year to year scientists tried to prove different theorems. As a result there left the Last Fermat's Theorem. Everyone tried to solve it, but no one succeeded. Paul Wolfsquel has commissioned 100,000 marks to the one who will prove this theorem in the next 100 years (from September 13, 1907).

Only 358 years after the theorem was created, it was proved by a British mathematician Andrew Wiles. Today, it is considered one of the most difficult tasks of humanity. Its proof takes 130 pages and can be understood by only 1000 people around the world.

In 358 years, as soon as they tried to prove this theorem, mathematicians suggested various statements referred to it and changed it for the easier proof. One of these tasks is mentioned in the book of Hugo Steinhaus "Hundred Tasks" on page 10 [2]:

$x^n+y^n=z^n$, $x, y, z, n \in N$, $n \geq z$.

Let's consider it.

If x, y, z, n are natural numbers, $n \geq z$, then the equation $x^n+y^n=z^n$ has no solutions.

Reasoning:

Let's assume that there are natural numbers x, y, z, n such as $n \geq z$ and $x^n+y^n=z^n$.

$x \neq y$, because if $x = y$ then:

$$x^n+x^n=z^n \rightarrow 2x^n = z^n \rightarrow x^n = \frac{z^n}{2} \rightarrow x = \frac{z}{\sqrt[n]{2}}$$

This means that the number x is irrational, which is contrary to the condition.

It can be seen that $x < z$, $y < z$ and $x < y$ or $x > y$.

Let's take $x < y \rightarrow z < y < x$. If all numbers x, y, z are minimal:

$x_{min} = 1$, $y_{min} = 2$, $z_{min} = 3$.

Then it is clear that $z - y \geq 1$.

$z > x \rightarrow z^{n-1} > x^{n-1}; z^{n-2} > x^{n-2} \ldots$

$y > x \rightarrow y^{n-1} > x^{n-1}; y^{n-2} > x^{n-2} \ldots$

$z^n - y^n = (z-y)(z^{n-1} + yz^{n-2} + \ldots + y^{n-1}) \geq 1 \times (x^{n-1} + xx^{n-2} + \ldots + x^{n-1}) = x^{n-1} + x^{n-1} + \ldots + x^{n-1}$

Since x^{n-1} in the inequality will be n items and $n \geq z > x \rightarrow n > x$, then:

$z^n - y^n \geq n\, x^{n-1} > x\, x^{n-1} = x^n$

$z^n - y^n > x^n$

And this equation contradicts the assumption that $x^n+y^n=z^n$. That is, the statement is true.

References

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2. Штейнгауз Г. Сто задач. – М.: Наука, 1951. – 144 с.