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EULER'S LINE AND EULER'S CIRCLE

Mathematics is an inexhaustible source of interesting tasks and paradoxes. We all know that in ancient Greece were made almost all major geometric discoveries, but today you can find such topics that require research. For example, such a figure as a triangle was the subject of the research of the famous mathematician Euclid, who found all the main properties of this figure. But in the 18th century the well-known mathematician Leonard Euler formulated important theorems connected with two new elements in a triangle – the Euler's line and the Euler's circle (or a circle of nine points). Let's discover these two theorems.

Before the formulating theorems, we give the definition of some points in a triangle. In fact, triangles have over 1602 different triangle centers. But the most important ones are centroid, incenter, circumcenter and orthocenter. Centroid can be found as the intersection of the three medians of the triangle. The centroid is also known as a center of mass. The incenter is the point where the internal angle bisectors of the triangle cross. The circumcenter is the center of a triangle's circumcircle and it can be found as the intersection of the perpendicular bisectors. The orthocenter is the point of intersection of three altitudes in triangle.
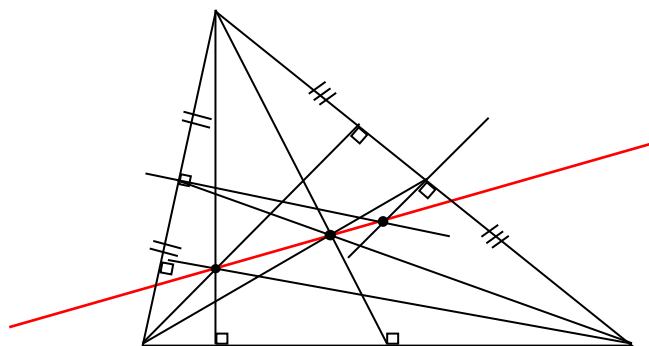

Euler showed in 1765 that in any triangle, the orthocenter, circumcenter and centroid are collinear or belongs to line which is called the Euler line. The fact that

such a line exists for all triangles is quite unexpected, made more impressive by the fact that the relative distances between the triangle centers remain constant. The interesting fact is that these nine points are also known as remarkable points of triangle.

The next theorem is the theorem about Euler`s circle or as it is also called the nine- point circle. This circle of a triangle is a circle going through 9 key points: the three midpoints of the sides of the triangle, the three feet of the altitudes of the triangle and the three midpoints from the vertices to the orthocenter of the triangle.

Sometimes the nine-point circle is referred to as the Feuerbach circle. The fact that the nine-point circle is tangent to the inscribed circle and the three escribed circles is Feuerbach's theorem.
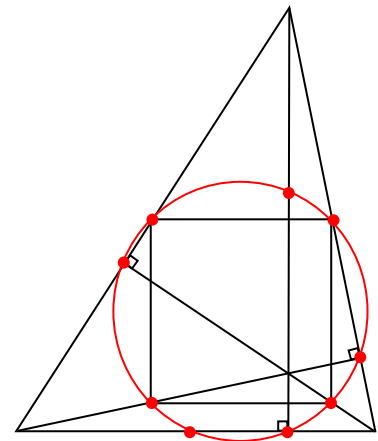
One of the most important property of a nine-point circle is that the radius of the nine-point circle is exactly half the circumradius of the triangle. Furthermore, due to symmetry, we have the following: the nine-point circles of triangles $ABC, HAB,$ BHC and CAH are all the same; points A_2, B_2, C_2 are circumcenters of triangles $AB_1C_1,$ $BA_1C_1,$ CB_1A_1 (AH, BH, CH are diametres of these circumscribed circles); the triangles $AB_1C_1,$ $BA_1C_1,$ CB_1A_1 are similar with each other and similar to triangle ABC; the points which are symetric to orthocenter comparatively to the sides of triangle belong to circumscribed circle of this triangle; the center of a nine-point circle belongs to the Euler's line.

In conclusion, these theorems proved that despite the huge number of discoveries which were made and well-known theorems which were proved, there will always be something that can be interesting for each of us and really amazing as for mathematicians.

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