E.A. SEVOST'YANOV, A.A. MARKYSH

On Sokhotski–Casoratti-Weierstrass theorem on metric spaces

Zhytomyr I. Franko State University, Zhytomyr, Ukraine
E-mail: esevostyanov2009@gmail.com, tonya@bible.com.ua

Let (X, d, μ) and $(X^{\,\prime}, d^{\prime}, \mu^{\prime})$ be metric spaces with metrics d and $d^{\prime}$ and locally finite Borel measures μ and $\mu^{\prime}$, correspondingly. Let us consider condition $\mathbf{A}$: for all $\beta : [a,\, b) \rightarrow X^{\,\prime}$ and $x \in f^{\,-1}\left(\beta(a)\right)$, a mapping $f : D \rightarrow X^{\,\prime}$ has a maximal f-lifting in D starting at x.

We say that a function $h : \overline{X} \times \overline{X} \to \mathbb{R}$ meets the requirement **B** on $\overline{X} := X \cup \infty$, if the following conditions hold:
$\mathbf{B}_1$: h is a metric on $\overline{X}$;
$\mathbf{B}_2$: $(\overline{X}, h)$ is a compact metric space;
$\mathbf{B}_3$: $h(x, y) \leqslant d(x, y)$ for every $x, y \in X$.

A mapping $f : G \setminus \{x_0\} \to G'$ is a *ring Q-mapping at a point $x_0 \in \partial G$ with respect to (p, q)-moduli*, if the inequality $M_p(f(\Gamma(S_1, S_2, A))) \leqslant \int\limits_{A\cap G} Q(x)\eta^q(d(x, x_0))\, d\mu(x)$ holds for each ring $A = A(x_0, r_1, r_2) = \{x \in X : r_1 < d(x, x_0) < r_2\}$, $0 < r_1 < r_2 < \infty$ and every measurable function $\eta : (r_1, r_2) \to [0, \infty]$ with $\int\limits_{r_1}^{r_2} \eta(r)dr \geqslant 1$.

Theorem. *Let $2 \leqslant \alpha, \alpha' < \infty$, $1 \leqslant q \leqslant \alpha$, $\alpha' - 1 < p \leqslant \alpha'$ and let (X, d, μ) be locally compact Ahlfors α-regular metric space. Let (X', d', μ') be Ahlfors α'-regular proper path connected, locally connected metric space where $(1; p)$-Poincaré inequality holds. Let $G := D \setminus \{\zeta_0\}$ be a domain in X, which is locally path connected at $\zeta_0 \in D$. Assume that $Q \in FMO(\zeta_0)$, and there exists a function h satisfying conditions **B**.*

*If an open discrete ring Q-mapping $f : D \setminus \{\zeta_0\} \to X$ at ζ_0 with respect to (p, q)-moduli satisfies the condition **A** and ζ_0 is an essential singularity of f, then $f(V \setminus \{\zeta_0\})$ is dense in X' for an arbitrary neighborhood V of ζ_0.*
