On equicontinuity of generalized quasiisometries on Riemannian manifolds

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Everywhere further D is a domain on a Riemannian manifold $(\mathbb{M}^n, g)$, $n \geq 2$, g is a Riemannian metric on $\mathbb{M}^n$ and v is a volume on $\mathbb{M}^n$, as well. Let (X, μ) be a metric measure space and let $1 \leqslant p < \infty$. We say that X admits *a* $(1; p)$*-Poincare inequality* if there is a constant $C \geqslant 1$ such that $\frac{1}{\mu(B)} \int\limits_B |u - u_B| d\mu(x) \leqslant C \cdot (\operatorname{diam} B) \left(\frac{1}{\mu(B)} \int\limits_B \rho^p d\mu(x) \right)^{1/p}$ for all balls B in X, for all bounded continuous functions u on B, and for all upper gradients ρ of u, $u_B := \frac{1}{\mu(B)} \int\limits_B u \, d\mu(x)$. Metric measure spaces where the inequalities $\frac{1}{C} R^n \leqslant \mu(B(x_0, R)) \leqslant CR^n$ hold for a constant $C \geqslant 1$, every $x_0 \in X$ and all $R < \operatorname{diam} X$, are called *Ahlfors n-regular.* We write $\varphi \in FMO(x_0)$, if $\overline{\lim\limits_{\varepsilon \to 0}} \frac{1}{v(B(x_0, \varepsilon))} \int\limits_{B(x_0, \varepsilon)} |\varphi(x) - \varphi_\varepsilon| dv(x) < \infty$, $\varphi_\varepsilon := \frac{1}{B(x_0, \varepsilon)} \int\limits_{v(B(x_0, \varepsilon))} \varphi(x) \ dv(x)$.

Theorem. *Let $p \in [n-1, n]$ and $\delta > 0$, and let a Riemannian manifold $\mathbb{M}^n_*$ be a connected Ahlfors n-regular space. Assume that $\mathbb{M}^n_*$ supports $(1; p)$-Poincare inequality. Let $B_R \subset \mathbb{M}^n_*$ be a fixed ball of a radius R, and let $Q \colon D \to [0, \infty]$ be a measurable function. Denote $\mathfrak{R}_{x_0, Q, B_R, \delta, p}(D)$ a family of all open discrete (p, Q)-mappings $f \colon D \to B_R$ at $x_0 \in D$, for which there exists a continuum $K_f \subset B_R$ such that $f(x) \notin K_f$ for all $x \in D$ and, besides that,* $\operatorname{diam} K_f \geqslant \delta$. *Then $\mathfrak{R}_{x_0, Q, B_R, \delta, p}(D)$ is equicontinuous at $x_0 \in D$ whenever $Q \in FMO(x_0)$.*