On equicontinuity of generalized quasiisometries on Riemannian manifolds

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Everywhere further $D$ is a domain on a Riemannian manifold $(\mathbb{M}^n, g)$, $n \geq 2$, $g$ is a Riemannian metric on $\mathbb{M}^n$ and $v$ is a volume on $\mathbb{M}^n$, as well. Let $(X, \mu)$ be a metric measure space and let $1 \leq p < \infty$. We say that $X$ admits a $(1;p)$-Poincare inequality if there is a constant $C > 1$ such that

$$
\frac{1}{\mu(B)} \int_B |u - u_B| d\mu(x) \leq C \cdot (\text{diam } B) \left( \frac{1}{\mu(B)} \int_B \rho^p d\mu(x) \right)^{1/p}
$$

for all balls $B$ in $X$, for all bounded continuous functions $u$ on $B$, and for all upper gradients $\rho$ of $u$, $u_B := \frac{1}{\mu(B)} \int_B u d\mu(x)$. Metric measure spaces where the inequalities $\frac{1}{R^n} \leq \mu(B(x_0, R)) \leq CR^n$ hold for a constant $C \geq 1$, every $x_0 \in X$ and all $R < \text{diam } X$, are called Ahlfors $n$-regular. We write $\varphi \in FMO(x_0)$, if $\lim_{\varepsilon \to 0} \int_{B(x_0, \varepsilon)} \frac{1}{v(B(x_0, \varepsilon))} \int_{B(x_0, \varepsilon)} |\varphi(x) - \varphi_x| dv(x) < \infty$,

$$
\varphi_x := \frac{1}{v(B(x_0, \varepsilon))} \int_{B(x_0, \varepsilon)} \varphi(x) \ dv(x).
$$

Theorem. Let $p \in [n - 1, n]$ and $\delta > 0$, and let a Riemannian manifold $\mathbb{M}^n$ be a connected Ahlfors $n$-regular space. Assume that $\mathbb{M}^n$ supports $(1;p)$-Poincare inequality. Let $B_R \subset \mathbb{M}^n$ be a fixed ball of a radius $R$, and let $Q: D \to [0, \infty]$ be a measurable function. Denote $\mathfrak{R}_{x_0, Q, B_R, \delta, p}(D)$ a family of all open discrete $(p,Q)$-mappings $f: D \to B_R$ at $x_0 \in D$, for which there exists a continuum $K_f \subset B_R$ such that $f(x) \notin K_f$ for all $x \in D$ and, besides that, $\text{diam } K_f \geq \delta$. Then $\mathfrak{R}_{x_0, Q, B_R, \delta, p}(D)$ is equicontinuous at $x_0 \in D$ whenever $Q \in FMO(x_0)$. 