On removable singularities of mappings in uniform spaces

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In what follows, (X, d, μ) and (X', d', μ') are metric spaces with metrics d and d' and locally finite Borel measures μ and μ', correspondingly; M_p denotes the p-modulus of a family of paths. Let $\overline{X} := X \cup \{\infty\}$, and let $h : \overline{X} \times \overline{X} \to \mathbb{R}$ be a metric. We say that h satisfies the *weak sphericalization condition,* if $(\overline{X}, h)$ is a compact metric space while h and d generate the same topology on X. A metric space X is called a *space admitting a weak sphericalization*, if there exists a metric $h : \overline{X} \times \overline{X} \to \mathbb{R}$ satisfying the weak sphericalization condition. Given $p \geqslant 2$, a space $\overline{X}$ is called *p-uniform* if, for each $r > 0$, there is $\delta = \delta(r) > 0$ such that $M_p(\Gamma(F, F^*, \overline{X})) \geqslant \delta$ whenever F and F^* are continua of $\overline{X}$ with $h(F) \geqslant r$ and $h(F^*) \geqslant r$. Given $2 \leqslant \alpha < \infty$ and $1 \leqslant q \leqslant \alpha$, the space $X = (X, d, \mu)$ is called (α, q)*-admissible source*, if (X, d, μ) be locally compact and locally path connected upper Ahlfors α-regular metric space, moreover, for each point $x_0 \in X$ there is $\gamma > 0$ such that

$$\mu(B(x_0, 2r)) \leqslant \gamma \cdot \log^{\alpha-2} \frac{1}{r} \cdot \mu(B(x_0, r)) \qquad (1)$$

for some $r_0 > 0$ and for all $r \in (0, r_0)$. Similarly, given $p \geqslant 2$, the space $X' = (X', d', \mu')$ is called *p-admissible target*, if (X', d', μ') admits a weak sphericalization, besides that, $\left(\overline{X'}, h\right)$ be locally connected p-uniform metric space.

Theorem 1. *Fix $2 \leqslant \alpha < \infty$, $2 \leqslant p < \infty$ and $1 \leqslant q \leqslant \alpha$. Let D be a domain in X, let (X, d, μ) be an (α, q)-admissible source and let (X', d', μ') be an p-admissible target. Suppose that $G := D \setminus \{\zeta_0\}$ is a domain in X, which is locally path connected at $\zeta_0 \in D$, $Q \in FMO(\zeta_0)$ and that balls $B_h(A, r) = \{y \in \overline{X'} : h(y, A) < r\}$ do not degenerate into points for each $A \in \overline{X'}$ and every $r > 0$. If $f \colon D \setminus \{\zeta_0\} \to X'$ is an open discrete ring Q-mapping with respect to (p, q)-moduli at ζ_0, and ζ_0 is an essential singularity of f, then $f(U \setminus \{\zeta_0\})$ is dense in X' for an arbitrary neighborhood U of ζ_0.*