On removability of isolated singularities of homeomorphisms with the inverse Poletsky inequality

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In what follows, M denotes the n-modulus of a family of paths, and the element $dm(x)$ corresponds to a Lebesgue measure in $\mathbb{R}^n$, $n\geqslant 2$. For given sets E and F and a given domain D in $\overline{\mathbb{R}^n}=\mathbb{R}^n\cup\{\infty\}$, we denote by $\Gamma(E,F,D)$ the family of all paths $\gamma:[0,1]\rightarrow \overline{\mathbb{R}^n}$ joining E and F in D, that is, $\gamma(0)\in E$, $\gamma(1)\in F$ and $\gamma(t)\in D$ for all $t\in [0,1]$. Let $x_0\in\overline{D}$, $x_0\ne\infty$,

$$S(x_0,r) = \{ x\,\in\,{\mathbb R}^n : |x-x_0|=r\}\,, S_i=S(x_0, r_i)\,,\quad i=1,2\,,$$

$$A=A(x_0, r_1, r_2)=\{ x\,\in\,{\mathbb R}^n : r_1<|x-x_0|<r_2\}\,.$$

Let $Q:{\mathbb R}^n\rightarrow {\mathbb R}^n$ be a Lebesgue measurable function satisfying the condition $Q(x)\equiv 0$ for $x\in{\mathbb R}^n\setminus D$. The mapping $f:D\rightarrow \overline{{\mathbb R}^n}$ is called a *ring Q-mapping at the point $x_0\in \overline{D}\setminus \{\infty\}$*, if the condition

$$M(f(\Gamma(S_1, S_2, D)))\leqslant \int\limits_{A\cap D} Q(x)\cdot \eta^n(|x-x_0|)\, dm(x) \tag{1}$$

holds for all $0<r_1<r_2< d_0:=\sup\limits_{x\in D}|x-x_0|$ and all Lebesgue measurable functions $\eta: (r_1,r_2)\rightarrow [0,\infty]$ such that $\int\limits_{r_1}^{r_2}\eta(r)\, dr\geqslant 1\,.$

Theorem 1. *Let D and $D^{\,\prime}$ be domains in $\overline{{\mathbb R}^n}$, $n\geqslant 2$, and let g be a homeomorphism of a domain $D^{\,\prime}$ onto a domain D, the inverse $f=g^{\,-1}$ of which satisfies the condition (1) at every point $x_0\in \partial D$. If $Q\in L^1(D)$ and y_0 is an isolated point of the boundary of the domain $D^{\,\prime}$, then the mapping g has a continuous extension $\overline{g}:D^{\,\prime}\cup\{y_0\}\rightarrow\overline{{\mathbb R}^n}$ to y_0.*