# GRAPHIC CULTURE OF CONSTRUCTIVE MODELING OF FIGURES IN METRIC STEREOMETRY

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**Abstract**. Two approaches for drawing of stereometric objects by a constructive method are presented using algorithms designed to help solve problems in the construction of correct, visual and easy-to-draw drawings. The first approach is based on operations borrowed from engineer drawing, where standardized axonometric projections guarantee the quality of drawings. The second one is based on the experience of many generations of teachers and practitioners. A list of methodological advice has been proposed. The analytical method of reasoning in computational problems is promoted, the method of combining with the image plane is established, which visualizes the way to the result, adds empiricism to the discipline. The presented statistical survey among mathematics teachers demonstrates their attitude to the constructive solution of stereometric problems. The advantages and disadvantages of teaching constructivism methods are highlighted. It has been established that the skills of qualitative drawings contribute to the formation of life competencies of pupils.

*Keywords:* methodology of geometry, constructivism of geometrical problem, graphic culture, modeling, alignment technique, visualization and visual representation.

## 1.Introduction

Working with first-year university students, it is easy to see that mathematics teachers do not teach pupils of 10-11 grades to create drawings of stereometric objects correctly. This can be easily explained by the following: the content of the educational program of the school mathematics course does not include lessons dedicated to the culture of drawings; pupils don't solve the problems using constructive methods; they do not know the theory for transformations of figures in space and have insufficient practice in supplementing of the drawings and their transformations.

This situation leads to the fact that the school graduate is not aware of the methods of modeling stereometric situations, and, therefore, is not table to solve problems of at least an average complexity degree. Students' logical thinking, spatial representations and imagination are not developed enough. Applied nature of the subject "Geometry" is lost. The young person, entering adulthood witrout the ability to think eye-minded in spatial categories and to implement his own thoughts (projects) with high-quality images, will not be able to adequately perceive the world around him, which reduces the opportunity to proper self-express in the society.

When teaching geometry, the teacher must emphasize three features of the decisions that should be observed: to be *correct*, *visual*, and *simple*. And this is valid especially for the modeling of the tasks by their visualization on the image plane, which could be a board, a notebook or a computer screen.

Since the image on the drawing plane is a result of parallel projection of an imaginary spatial object, the student must to know the properties of this operation and follow them, which will guarantee the correct result. Visuality means that anyone, having entered the room and looked at the drawing, understands what kind of a figure is depicted on it. Simplicity in construction is when a teacher or student, modeling some stereometric object in compliance with the first two requirements, works quickly, but always with high quality.

It is important to carry out a preliminary analysis of the statement of the task, planning the creation of a qualitative drawing. Working with a drawing, its transformations, which contribute to the algorithmizing of the solution, considerably depend on the thoroughness of the analysis of the condition and the chosen angle on an imaginary object in the parallel projection, rational, successful figure positioning between the observer's eye and the projection plane.

At the end of the 19th century, the great German geometer D. Hilbert (1862–1943) noted: "In Mathematics, as in scientific researches, there are two trends: the tendency to abstraction – it tries to develop a logical point of view based on various material and bring all this material into a systematic connection, and another tendency, the tendency towards **visualization**, which, in contrast, strives forward a **living understanding** of objects and their internal relations"(Gilbert 1981).

The role of visualization in teaching was described by the well-known geometer Aleksandrov O.D., the mentor of the outstanding Ukrainian geometer Pogorelov O.V.: "The geometric method does consist in the fact that the logical proof itself or the solution of the task is guided by a visual representation; it is the best when the proof or solution can be seen from a drawing. (In ancient Indian writings, it happened that the proof was reduced to a drawing signed with one word "Look!"). The student should be accustomed to the same approach - to start with a drawing, a sketch, a visual description - does not matter whether he is examined in the front of the blackboard, whether he learns something at home, whether he solves a task. Along with the drawing there should be a spatial representation, an accurate understanding, etc. Concepts coming from visual Geometry are, in general, extremely important in modern science, so one should not think that visual is lower, and not Higher Mathematics. From the simple and visual there is a way to the higher - the way of Geometry" (Aleksandrov 1980).

The issues of creativity and the peculiarities of the use of visual literacy, the use of spatial representations and imagination for simulated images in studying and life, the culture of drawing and the expression of opinions with high-quality drawings are the subject of research of many leading teachers (Boden&Stenliden 2019; Kremen&Iljin 2020; Koval&Besklinskaya 2020).

Well-known Ukrainian scientists and teachers (Astriab et al. 1956; Slepkan 1983; Mikhailenko&Teslenko 1965) paid considerable attention to creating drawings and the visualization in Stereometry.

A significant assistant in the modeling of geometric figures is modern computer with reliable software and the pedagogical tools, which should be meaningfully used in educational institutions (Botana et al. 2014; Anishchenko et al. 2021; Bykov et al. 2020; (Lenchuk and Shchekhorskyi 2021), because "The use of a computer should be pedagogically balanced and appropriate" (Zheldak 2011).

Our textbook for students and teachers contains the basics of teaching Stereometry by constructive method (Lenchuk 2010). The methodology of a systematic approach to the formation of competencies for performing drawings of spatial objects, solving positional and metric problems by construction methods are considered.

The **constructive method in Geometry** is the trend according to which each geometric object or statement about it should be the result of a mental activity that uses transformations and visual images in accordance with simple and easily checked rules, i.e. algorithms. With their help, through a finite number of steps and operations the expected construction is obtained for a certain time.

## 2.Guidelines

Many years of experience in teaching Geometry shows us that in the process of teaching a high-quality drawing of three-dimensional figures and their combinations, it is useful to adhere to the following rules:

1. Perform graphic operations on the drawing field with solid thin lines initially (fig. 1), and then redraw them with standard lines: the visible outlines - with a solid base line; the invisible - with dashed; the central, axial and symmetry lines - with a thin chain-dotted line.

2. For achieving the necessary effect in the redrawing use pencils of different hardness: 2B, B, HB, F, H, 2H.Working "by hand", it is enough to have pencils of hardness 2B and F, neatly sharpened.

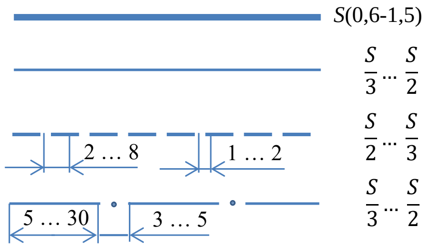



**Figure 1.** Standard “lines”

3. Drawing a straight line from point to point, focus your look on the end point. Let one of the fingers holding the tool slide along the image plane. The technique is useful when the drawing is done on a blackboard with a piece of chalk.

4. Start drawing a prism (cylinder) from the upper base, because the additional constructions will be connected with the lower base of the figure. Choose the vertex of the pyramid carefully so that the visible side edges do not obscure the invisible vertical height.5. Position the object relative to the drawing plane so that the largest number of faces is visible.

6. When drawing a combination of two figures do three steps: 1) create a drawing of the surface of one of the figures; 2) find the common elements of both surfaces (contact points, lines) on it; 3) create the drawing of the other figure so that the elements found in 2) belong to its surface.

7. When working on drawings of combinations of two figures, keep in mind that the surface of the described figure must be transparent with respect to the inscribed one, and each of the surfaces is opaque with respect to itself.

8. Creating drawings with the constructive method, use detail drawings so as not to clutter up the image.

9. To make the drawing more clear and the visual perception of the dependencies between the parts of the figures better use the following elements: distorted and original right angles; if necessary, indicate the degree measure of the angle; to stress the nuances of the statement of the task and see better the connections between the elements, designate the equal segments with the same number of strokes; to better represent the shape of the figure use colored pencils.

10. Remove extra lines of drawings with a soft eraser.

11. The key to solving a particular type of Stereometry problem lies “on the surface” if you know how to operate with the complex drawings by G. Monge.

12. It is useful to understand the essence and acquire practical skills in performing drawings using the method of axonometric directions and conditional relationships. This approach will allow you to create computer programs independently and use software pedagogical teaching aids effectively.

Following the above guidelines guarantees the correctness and clarity of the drawings. These drawings will be simple in construction if the subject of teaching receives practical modelling skills. Aesthetically attractive, accurate projection drawings will contribute to the visualization of reasoning, rational and clear algorithm presentation of problem solving, interest in constructive operations and self-confidence.

## 3. Examples

Let’s solve several problems of a constructive-calculation nature.

**Problem 1**. *In a regular triangular pyramid SABC, the height is equal to the side a of the base. Construct a section of the pyramid by a plane passing through the base edge AB, perpendicular to the edge SC. Calculate the area of the section figure and the ratio of the volumes of the parts into which the section divides the pyramid.*

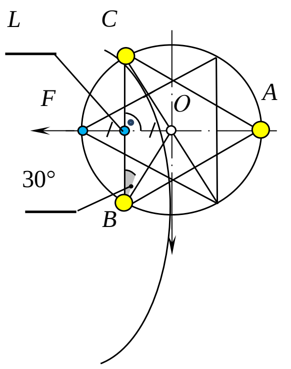



**Figure 2.** Regular triangle

In a triangle, it is easy to introduce conditional (approximate) ratios. Every student knows how to inscribe a regular triangle *ABC* in a circle centered at the point *O* (fig. 2). Here it is convenient to take the circle radius R equal to 6 un.m.

**Добавлено примечание ([KM1]):** Explain whit is the goal of this paragraph concerning the solved task? Why is the figure so complex: points L, F, second triangle arc of some bigger circle, some angle of 30 degrees - if they are necessary for the solution, if not - draw simple figure

The algorithm for modeling a pyramid with a triangle *ABC* as a base is the following (figures 3.a; 8.b, d):

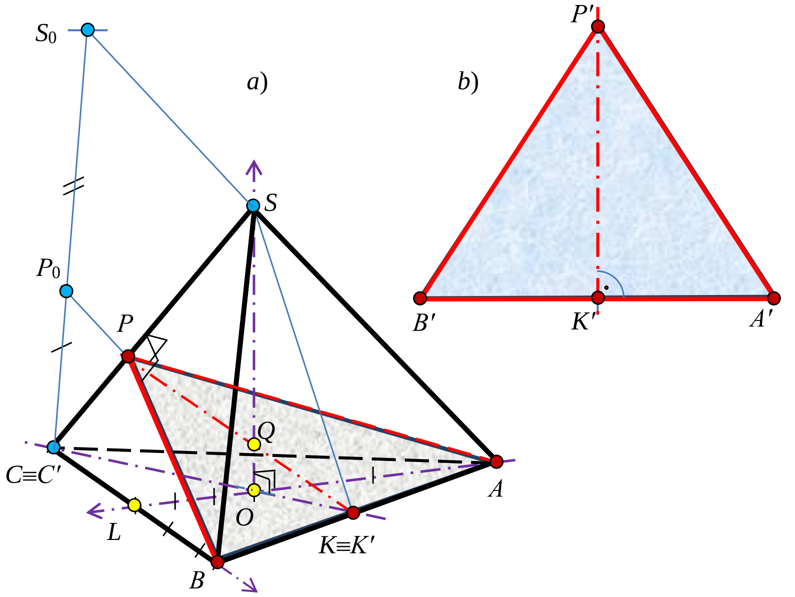
1) draw a straight line at an angle of 7°10′ to the horizon and select point *O*, as a center of the triangle, on it;

2) from the point *O* lay down three one unit segments, denoting the end point of third with *L* (length of *OL* = 3) and draw a straight line through the point *L*, inclined to the horizon at an angle of 41°25′;

3) in both directions of the drawn straight line, set aside 2.5 units segments, defining in such a way the two vertices *B* and *C* of the triangle;

4) from the point *O* upwards, lay off the segment *OA*, twice as long as the segment *OL* (*OA* = 6), defining the third vertex of the triangle *A*;

5) the height of the pyramid is placed in *O* vertically, depicted with a chain-dotted thin line.




**Figures 3 a, Figure 3 b.** Construction of plane section of a pyramid

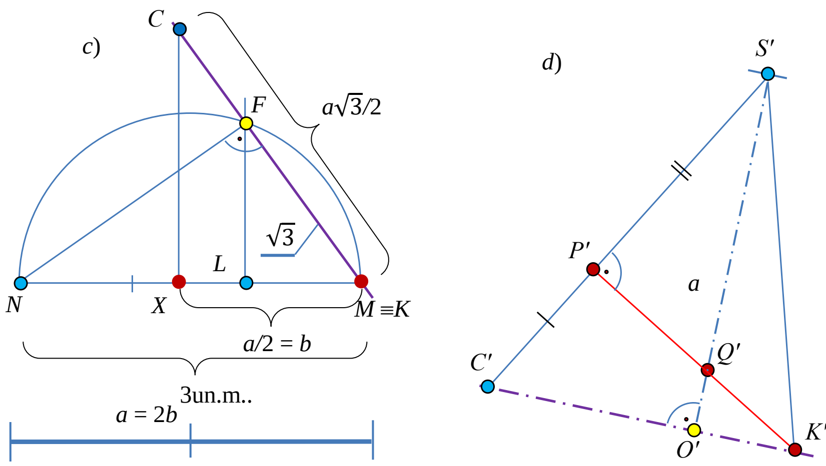
**Добавлено примечание ([KM2]):** There are two Figure 3 - here a. and c. and in the other b. and d. Let here to be 3.a and 3.b and the other to be with other number F - F.a and F.b

Algorithm explanation. Triangle *BLO* is right-angled (∠*BLO* = 90°), radius *BO* of the described circle with center *O* bisects angle *CBA*: ∠*LBO* = 30°; tg 30° = $\frac{OL}{LB}$≈ 0,5774 ≈ 0,6= $\frac{3}{5}$ ($\frac{3}{5}$ is the defining relation in the right triangle for its visual construction).

Problem analysis. The edges *SC* and *AB* are mutually perpendicular by the *Three perpendiculars* theorem. Therefore, it is enough to draw a perpendicular *KP* from the point *K*, such that *AK* = *KB*, to the side edge *SC*, which will determine the section plane **Σ** (*KP* × *AB*) ⊥ *SC*.

Moving the pyramid *SABC* in space, combine it with the picture plane by the median $CK \equiv C'K'$ and select the segment $C'K' = \frac{a\sqrt{3}}{2}$ as original. By rotation around the line $C'K'$ of the zero level, “put” the triangle *SKC* onto the image plane. To find the location of the point *S′,* the image of the point *S*, modeling the true length of the height of the pyramid $S'O' = a$ by performing the following operations.

**Добавлено примечание ([KM3]):** Here the points with primes appear without definitions and we are not able to check more ...




**Figures 3 c, 3 d.** The process of section construction

First, build a segment $b$ such that $C'K' = b\sqrt{3}$, where $b = \frac{a}{2}$, and secondly, take two segments $b$ (Figure 3, c). In the right triangle *MFN*, the geometric mean is a leg *MF* with hypotenuse $MN = 3$ un.m., and $KS = K'S'$ is marked on the ray *MF*. Obviously, the triangles *MCX* and *MFL* are similar, whence: $\frac{MX}{ML} = \frac{MC}{MF}$. But $ML = 1$ and $MF = \sqrt{3}$. Thus, $MX = \frac{KC}{\sqrt{3}} = = b = \frac{a}{2}$. Using the segment $a$, combine the triangle $S'C'K'$ with the image plane (Figure 3, *d*) and drop the perpendicular $K'P'$ from the point $K'$ to the side $S'C'$. Dividing the edge of the pyramid *SC* with the point *P* in the ratio in which the point $P'$ divides the segment $S'C'$ and connecting the points *A-P-B* (Figure 3, *a*), obtain a section of the pyramid by a plane. The true shape and size of the cross-section is modeled as triangle $A'P'B'$ with base $A'B' = a$ and height $K'P'$ (Figure 3, b).

Calculations ($S = \frac{3}{8}a^2$ – the area of basis of a pyramid, $V_1 : V_2 = 5:3$) and an assessment of the accuracy of the constructions are proposed to be carried out independently.

**Problem 2**. *In a rectangular parallelepiped $ABCDA_1B_1C_1D_1$, with the given ratio of edges $AB : AD : AA_1 = 1 : 2 : 1$, a plane is drawn through vertices B, $C_1$ and D. Draw the perpendicular from point P on the edge $A_1D_1$, such that $A_1P : PD_1 = 1 : 2$ to the plane $BC_1D$.*

For high-quality modeling of a rectangular parallelepiped by a drawing, we will depict it with a rectangle at its base. According to the properties of parallel projection, a rectangle, or its varieties, is depicted in the figure by an arbitrary parallelogram. Therefore, the rectangle *ABCD* is depicted as follows (fig. 4.a):

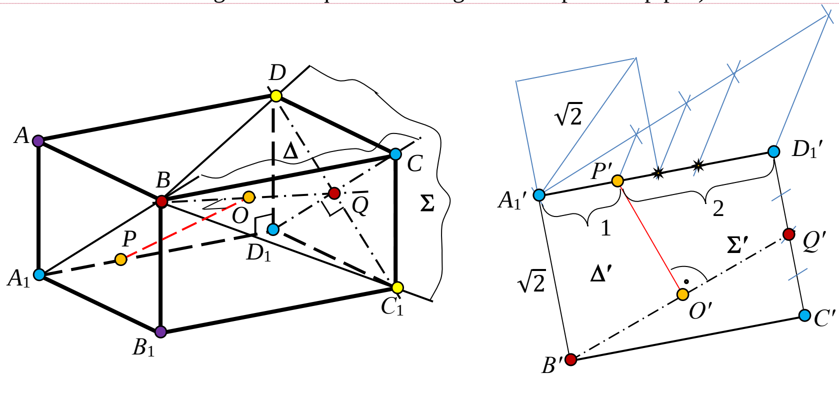
1) with an inclination of 10-15° to the horizon, draw a straight line and mark the side *BC* on it;

2) draw a ray from point *B* approximately at an angle of 120° to *BC* and mark a segment BA ≈ ½ BC on it;

3) complete the triangle *ABC* to parallelogram;

4) since the parallelepiped is rectangular, transfer the parallelogram *ABCD* trough the vector $\overrightarrow{AA_1}$ vertically down to parallelogram $A_1B_1C_1D_1$.

Analyzing the statement of the task and the figure, it is noticed that the planes **Δ** ($A_1BCD_1$) and **Σ** ($BC_1D$) are perpendicular (a criterion of perpendicularity of two planes), and the diagonal $DC_1$, belonging to the plane **Σ** and the face $DCC_1D_1$, is perpendicular to the two straight lines of the plane **Δ**: $DC_1 \perp A_1D_1$, $DC_1 \perp CD_1$. The planes intersect along the straight line BQ (where Q is a point of intersection of diagonals of square in the right face of parallelepiped).




*a*) *b*)

**Figure 4.** Distance from the Point to the Plane

By rotating around the straight line of the zero level, "put" on the drawing (Figure 4, *b*) the rectangle $A_1BCD_1$, where choose the original segment $A_1D_1 = A_1'D_1' = 2$ un.m. Note that the blue triangle in Fig. 4 b shows the division of the segment A1D1 in the ratio 1: 2 and therefore does not require designations. Simple operations build the true distance from the point $P'$ to the line $B'Q'$ (P′ – the image of the point P in the combination transformation, and where Q′ is the image of the point Q in the matching transformation; other pairs of points - similarly).

*BQ* is divided in the ratio $BO : OQ = B'O' : O'Q'$ (Figure 4, *a*).

**Problem 3**. *A prism is based on a regular triangle ABC. The two side faces of the prism are rhombus with a common edge $AA_1$ and an acute angle of* 60°*. Drop a perpendicular from point P (AP: $PA_1$ =* 1: 1*) of edge $AA_1$ to diagonal $BC_1$ of face $BB_1C_1C$.*

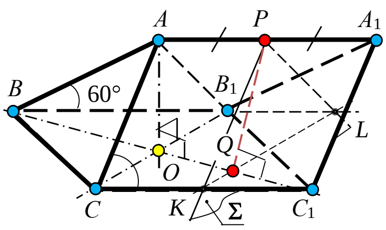



**Figure 5.** Distance from the Point to the Line

A high quality drawing is the main tool in the process of analyzing the conditions of the problem and creating an algorithm for solving it, the angle for placing the prism in space should be successfully chosen. The acute angles of two rhombus with a common edge $AA_1$ are equal to 60°, and their diagonals, opposite to these angles, divide the rhombus into two regular triangles and all the edges of the prism are equal. The side faces $AA_1C_1C$ and $AA_1B_1B$ are equal and inclined at equal angles to the plane of the base $A_1B_1C_1$ (*ABC*), and the third face of the $BB_1C_1C$ prism is a square. The heights of triangles $ABB_1$ and $ACC_1$ divide their bases $BB_1$ and $CC_1$ in half, triangle $ACB_1$ is right-angled and isosceles, and vertex *A* is projected into the center *O* of the square. Place the side face $BB_1C_1C$ of the prism at the base (Figure 5)

**Добавлено примечание ([KM4]):** For Q we have to guess which point denotes by the drawing. Please define these points in text - P', Q, Q', O, O' ….. And also triangle in blue, and blue square ….

**Добавлено примечание ([KM5]):** Here points with primes seems to be projections on plane of A1B1C1D1 but why P from statement of the task became P'?

and depict it a little differently: place the side $CC_1$ horizontally, the side $CB$ adjacent to it - approximately at an angle of 120° to $CC_1$ and half as much (this is the construction algorithm in Axonometry (rectangular dimetry).

Plane ($ACB_1$) $\perp BC_1$. Through the point $P$ of the edge $AA_1$draw the plane **Σ**($PKL$) parallel to ($ACB_1$). Parallel planes, crossed by a third plane, cut out parallel straight lines and simply draw the point $Q$-the base of the required perpendicular $PQ$.

The segment $PQ$ in its original size should be built using a remote drawing, aligning the triangle $PKL$ with the image plane, where $P'K' = P'L' = 1$, a $KL = CB_1 = K'L' = \sqrt{2}$.

**Problem 4**. *A plane is drawn through the side of the base of a regular quadrangular pyramid, cutting off a triangle with an area of* 4 *cm*$^2$ *from the opposite face. Find the lateral surface of the pyramid, cut off by a drawn plane, if the lateral surface of this pyramid is* 25 *cm*$^2$.

Modeling the drawing-picture of a square in a slightly different way – according to the rules of a rectangular dimetry - (Figure 6): 1) draw a straight line at an angle of 7°10′ to the horizon and select point $O$ on it - its center; 2) from the point $O$ to the left and to the right mark equal segments($OA = OC$); 3) at point $O$ draw a straight line inclined to the horizon at an angle of 41°25′; 4) mark the segments$OB = OD = \frac{OA(OC)}{2}$on this straight line from the point $O$. Thinking analytically further.

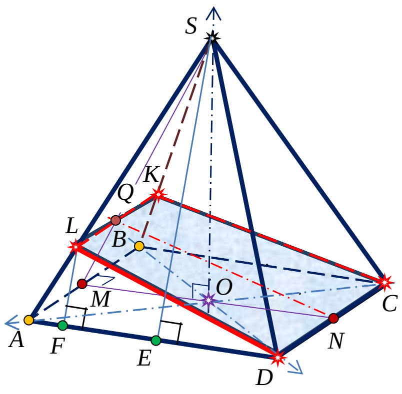


**Figure 6.** Model of a calculation problem

Calculate the area of the lateral surface of the pyramid $SKLDC$, containing four triangles:$SKL$ (the area is known), $SDC$ ($S_{\Delta SAB} = S_{\Delta SDC}$), $SLD$and$SKC$ (the triangles are equal, which is obvious). Let's complete the drawing in triangles$SAD$ and$LAD$:$SE = SM$and$LF = QM$. Calculate the area of the triangle $SDC$,remembering that the side faces of the pyramid$SABCD$are equal, and there are four faces:$S_{\Delta SAB} = S_{\Delta SDC} = 25 : 4 = 6{,}25$ (cm$^2$). The areas of similar figures are related as the squares of their linear elements.This allows to calculate the height of the side face, which precedes the calculation of the areas of triangles $SLD$and$SKC$. Thus: $S_{\Delta SLK} : S_{\Delta SAB} = SQ^2 : SM^2 = 4 : \frac{25}{4} = = \frac{16}{25}$, and $\frac{SQ}{SM} = \frac{4}{5}$.

It is shown that $S_{\Delta SLD} = S_{\Delta SKC} = S_{\Delta SAD} - S_{\Delta LAD} = \frac{1}{2}AD \cdot SE - \frac{1}{2}AD \cdot LF = \frac{1}{2}AD \cdot (SE - LF) = = \frac{1}{2}AB \cdot (SM - QM) = \frac{1}{2}AB \cdot SQ = \frac{1}{2}AB \cdot \frac{4}{5}SM$, and $S_{\Delta SLD} = \frac{4}{5}S_{\Delta SAB} = \frac{4}{5} \cdot 6{,}25 = 5$ (cm$^2$). In total $S_{SKLDC} = 20{,}25$ cm$^2$.

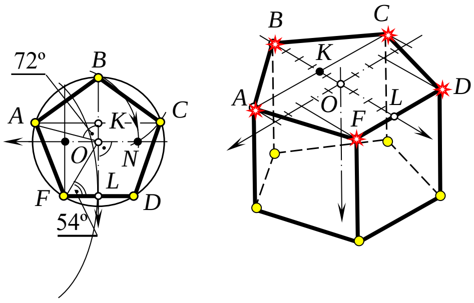


**Figure 7.** Right pentagon

Concluding the presentation of the text, the following should be noted separately.

The problems were not solved with figures that could be based on a regular pentagon. There are fewer of them, but they are in the books.

Assured the quality of the drawing, conditional ratios in a pentagon inscribed in a circle (Figure 7), calculating them as the lengths of the legs of right-angled triangles $OAK$ and $OLF$, where the hypotenuse is 5 un.m., and acute angles are 72°and

54°respectively were introduced. For example, $OK = AO \cdot \cos 72° \approx 5 \cdot 0{,}3090 \approx 1{,}5$ un.m.

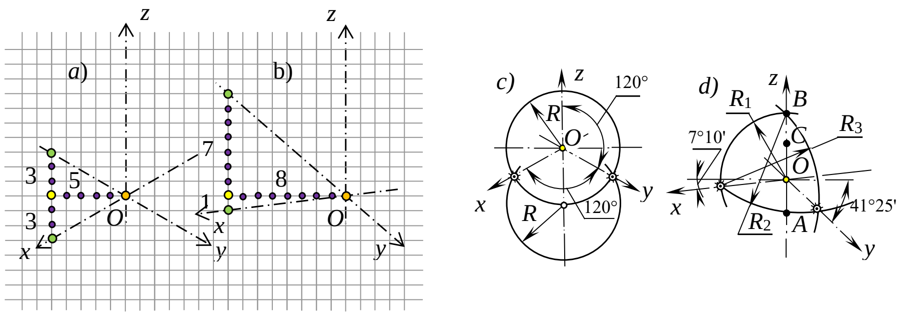



**Figure 8.** Axonometric Axis

Drawing construction algorithm (in isometric): 1) draw two straight lines at an angle of 120°to one another and equally inclined to the horizon (Fig. 8, *a*, *c*);2) draw the most distant vertex of the pentagon *B* (*OB* = 5 un.m.), located up and to the left of the point *O* on one of the lines drawn;3) draw a straight line parallel to another straight line through the point *K* on the straight line *OB* (*OK*=1,5 un.m.), mark *KA=KC*=4,75 un.m. on it; 4) from the point *O* to the right-down mark the segment *OL* = 4 un.m. and through the point *L* draw a straight line parallel to another straight line, mark the segments *LF=LD*=3 un.m. on it.

There is a piece of advice to the teacher, a student, a pupil: take into account that the described technique for a high quality drawing of a regular pentagon is the simplest.

**4. Description and results of the statistical experiments**

The program of the advanced training courses for teachers from Zhytomyr State University named after Ivan Franko stipulate the topics, which relate to the issues of solving multilevel stereometric tasks by constructive-computational methods. To find out the attitude of participants to this type of problem and to clear up the ways to solve them using constructive methods at school a questionnaire was worked out. Here are the questions of the questionnaire with the conclusive results and comments.

**1**. In your opinion, what is the role of drawing in Stereometry problems for calculation?

a) a correct and visual drawing is the main tool in finding a result;

b) drawing is important, but not the main thing in solving the task;

c) the quality of the drawing is not important.

**Comment:** Out of 127 respondents only 5 (3, 93%) chose the answer c), 34 (26.77%) – the answer b) , the rest (69.3%) – answered a) . Most of the teachers understand that a drawing is the primary tool of finding the solution of the task. Some mistakenly think that the drawing is irrelevant, but a poor-quality drawing can lead to false conclusions.

2. Do you accept the requirements for the quality of the drawings?

a) yes, I do;

b) I inform students that requirements exist, but I do not demand compliance with them;

c) I rely on the students' skills and experience;

d) do not accept them and do not comply in the learning process.

**Comment:** Point d) was not chosen by any course participant. 43 of them (33.86%) rely upon previously acquired skills and experience of participants; the majority of 68 participants (53.54%) chose answer b), and only 16 (12.6%) teachers justify the requirements for the quality of the drawings. A significant percentage of teachers hope that students from previous experience can build quality drawings. However, students should first be taught constructivism, and then be required to comply with the requirements for drawings.

3. Which method do you use in binary modeling of figures drawings and their combinations in Stereometry?

a) axonometric directions and conditional ratios;

b) experience of teachers and scientists;

c) you have your own experience for high-quality constructions.

***Comment:*** Significant majority of teachers – 80 (63%) – prefer their own experience; 32 participants (25.2%) use the experience of past generations of teachers and scientists, and only 15 (11.8%) master the method of axonometric directions and conditional ratios. Teachers are recommended to remember that it is possible to use a computer and software in teaching geometry effectively only if they master the method of axonometric directions and conditional relationships.

4. What constructive problems do you solve with students?

a) clearly demonstrating the conditions of tasks with high-quality drawings;

b) solving positional problems by construction methods;

c) modeling metric problems using constructivism methods: build involute surfaces of figures, find the areas of figures, distances from a point to a straight line (plane), measurement of angles, etc.

***Comment:*** Point c) was chosen by 9 (7.09%) respondents, the vast majority of teachers 118 (92.91%) combined points a) and b) explaining that in computational problems it is necessary to build drawings of separate figures, their combinations and sections of figures by a cutting plane. Not all teachers understand that, to a greater extent, practical (applied) geometry deals with metrics - measurements.

5. Do you want to improve students' competence in Stereometry by solving tasks using constructive methods?

a) yes, I would love to see that happen– 41 (32,3%);

b) yes, but I do not have enough time for these tasks– 64 (50,4%);

c) yes, if it was intended by the program– 7 (5,5%);

d) no, that's not the goal now– 9 (7,1%);

e) I don't think I need it– 6 (4,7%).

***Comment:*** The problem of time and programs in teaching pupils really exists. However, the teacher has to find opportunity to deepen their competence in constructive Stereometry in special or electives courses.

6. How the tasks requiring constructive approach promote students in their development?

a) contribute to improve the spatial representations and imagination;

b) contribute to improve the logic of reasoning;

c) contribute to the improvement of the analytical way of thinking;

d) demonstrate the applied nature of the discipline "Geometry";

e) they set students up for a more subtle understanding of the essence of things in the world.

***Comment:*** It is not surprising that the teachers (having listened to lectures on these topics) were unanimous, fully agreeing that constructive tasks are really those that promote.

7. What disadvantages of tasks that require a constructive approach would you mentioned? (a free answer question)

The most noticeable shortcomings that respondents mention was:

a) students' ignorance of the transformations of figures in space;

b) lack of experience in solving tasks by construction method;

c) students hardly understanding the process of graphic modeling the task;

d) lack of accuracy in the drawing performing;

e) significant duration of the process of solving such tasks;

f) inhibition of initiative and creative thinking in the step-by-step implementation of operations;

g) there is not enough methodological literature on the use of constructive methods in solving tasks of Stereometry.

The questionnaires of teachers regarding constructive-computational methods for solving problems in schools are carried out regularly, the results are correlated.

Добавлено примечание ([KM6]): Survey of what? explain

As a separate matter. The program for applicants of the first educational level (Bachelor degree) for future teachers of Mathematics contains the educational component "Selected questions of Geometry". The course consists of teaching students to use a constructive method for solving stereometric problems. The core content of the discipline has been formed over decades. The results of the final papers of participants testify to the effectiveness of the developed teaching methodology. The basic provisions of the research are systematized in our textbook (Lenchuk 2010).

**5. Conclusions**.

The outstanding French architect and architectural theorist Le Corbusier (1887-1965) wrote: "I think that we have never lived in such a geometric period. It is worth thinking about the past, remembering what was before, and we will be stunned to see that the world around us is a world of Geometry, pure, real, flawless in our eyes. **Everything around is Geometry**" (Le Corbusier 1970).

The results of the survey at the advanced training courses for teachers demonstrate that the school pays little attention to the culture of high-quality modeling of Stereometry objects, does not promote constructive methods for solving positional and metric problems. Teachers are poorly prepared to solve such tasks, they do not perfectly master the rules and techniques for transforming drawings, do not demonstrate the empiricism of a discipline, and do not contribute to the development of ideas, visual-figurative and logical thinking. All this impoverishes Geometry in the eyes of students.

In this paper, with a thorough analysis of the visualization aspects of projection drawings for the tasks of the first of the sciences, visualizing, proving by drawings, a constructive approach and an analytical method of reasoning on the way to a result, making out each of the problems with high-quality drawings, we offered to the student (pupil) the innovative methods of activity verified by practice in the course "Stereometry."

It is not difficult to explain the need to introduce elements of constructivism at school, because drawing is considered the main tools of teaching Geometry, and graphic and graph-analytical techniques for operations with various kinds of figures form the basis of the Applied Geometry discipline. Without it would be impossible to design and manufacture a modern aircraft, the machine-building and chemical industries, construction and architecture, light industry, etc. would not function without it. The main goal is to motivate the student and to prepare him for a full-fledged creative life. Geometry enriched with constructive methods will promote this goal.

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