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On the Dirichlet problem for degenerate Beltrami equations

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We study the Dirichlet problem $\lim\limits_{z \to \zeta} \operatorname{Re} f(z) = \varphi(\zeta)$ *as* $z \to \zeta$, $z \in D$, $\zeta \in \partial D$, *with continuous boundary data* $\varphi(\zeta)$ *in arbitrary simply connected bounded domains D of the complex plane* $\mathbb{C}$, *where f satisfies the degenerate Beltrami equation* $f_{\bar{z}} = \mu(z) f_z$, $|\mu(z)| < 1$, *a. e. in D. We give in terms of* μ *the BMO and FMO criteria as well as a number of other integral criteria on the existence and representation of regular discrete open solutions to the stated above problem.*

Keywords: *BMO, bounded mean oscillation, FMO, finite mean oscillation, Dirichlet problem, degenerate Beltrami equations, simply connected domains*

1. Introduction. Let D be a domain in the complex plane $\mathbb{C}$, and $\mu : D \to \mathbb{C}$ be measurable with $|\mu(z)| < 1$ a. e. in D. A *Beltrami equation* is an equation of the form

$$f_{\bar{z}} = \mu(z) f_z \tag{1}$$

with the formal complex derivatives $f_{\bar{z}} = (f_x + if_y)/2$, $f_z = (f_x - if_y)/2$, $z = x + iy$, where f_x and f_y are partial derivatives of f in x and y, correspondingly. The *dilatation quotient* of the equation (1) is the quantity

$$K_\mu(z) := \frac{1 + |\mu(z)|}{1 - |\mu(z)|}. \tag{2}$$

The Beltrami equation is called *degenerate* if $\operatorname{ess\,sup} K_\mu(z) = \infty$. The quantity

$$K_\mu^T(z, z_0) := \frac{\left|1 - \dfrac{\overline{z - z_0}}{z - z_0} \mu(z)\right|^2}{1 - |\mu(z)|^2} \tag{3}$$

is called the *tangent dilatation quotient* of (1). Note that

$$K_\mu^{-1}(z) \;\leqslant\; K_\mu^T(z, z_0) \;\leqslant\; K_\mu(z) \qquad \forall\, z \in D,\; z_0 \in \mathbb{C}. \tag{4}$$

The *Dirichlet problem* by [1] and [2] for the nondegenerate Beltrami equations (1) in a domain $D \subset \mathbb{C}$ is the problem on the existence of a continuous function $f : D \to \mathbb{C}$ in the Sobolev class $W_{loc}^{1,1}$, satisfying (1) a. e., such that, for each prescribed continuous function $\varphi : \partial D \to \mathbb{R}$,

$$\lim_{z \to \zeta} \operatorname{Re} f(z) = \varphi(\zeta) \qquad \forall\, \zeta \in \partial D. \tag{5}$$

Recall that mapping $f : D \to \mathbb{C}$ is called *discrete* if the preimage $f^{-1}(y)$ consists of isolated points for every $y \in \mathbb{C}$, and *open* if f maps every open set $U \subseteq D$ onto an open set in $\mathbb{C}$. Further, if $\varphi(\zeta) \neq const$, then the *regular solution* of the Dirichlet problem (5) for the Beltrami equation (1) is a continuous, discrete and open mapping $f : D \to \mathbb{C}$ of the Sobolev class $W_{loc}^{1,1}$ with its Jacobian $J_f(z) = |f_z|^2 - |f_{\bar z}|^2 \neq 0$ a. e. satisfying (1) a.e. and the condition (5).

If D is the unit disk, some criteria for the solvability of the Dirichlet problem for the degenerate Beltrami equation can be found in a monograph [3]. The case of Jordan domains have been studied e.g. in papers [4] and [5]. With the help of the concept of the Carathéodory prime ends, we have extended the above criteria to more general domains in [6]. However, this approach can be quite complex and difficult to apply in practice. The following proposition, together with appropriate existence theorems for the degenerate Beltrami equation in the whole complex plane, offers another approach to studying the Dirichlet problem in the classic setting (5) in *arbitrary bounded domains* $D \subset \mathbb{C}$. These results are provided in the following sections.

Proposition 1. *Let D be an arbitrary bounded simply connected domain in $\mathbb{C}$ and let $\mu : D \to \mathbb{C}$ be a measurable function with $|\mu(z)| < 1$ a. e. in D. Suppose that there exists a homeomorphic $W_{loc}^{1,1}(\mathbb{C})$ solution g to the Beltrami equation (1), with the same μ in D extended by zero outside of D. Then the Beltrami equation (1) has a regular solution f of the Dirichlet problem (5) in D for each continuous function $\varphi : \partial D \to \mathbb{R}$, $\varphi(\zeta) \neq const$. Moreover, such a solution f can be represented as the composition $f = h \circ g$, where $h : D_* \to \mathbb{C}$, $D_* = g(D)$, is a holomorphic solution of the Dirichlet problem*

$$\lim_{\xi \to \zeta} \operatorname{Re} h(\xi) = \varphi_*(\zeta) \quad \forall\, \zeta \in \partial D_*, \quad \text{with } \varphi_* := \varphi \circ g^{-1}.$$

Proof. First note that the domain D_* is also bounded. Then, by Theorem 4.2.1 and Corollary 4.1.8 in [7], there is a unique harmonic function $u : D_* \to \mathbb{R}$ that satisfies the Dirichlet boundary condition

$$\lim_{\xi \to \zeta} u(\xi) := \varphi_*(\zeta) \quad \forall\, \zeta \in \partial D_*, \quad \text{where } \varphi_* := \varphi \circ g^{-1}.$$

On the other hand, there is a conjugate harmonic function $v: D_* \to \mathbb{R}$ such that $h := u + iv : D_* \to \mathbb{C}$ forms a holomorphic function, because the domain D_* is simply connected, see e.g. arguments at the beginning of the book [8]. Thus, the function $f = h \circ g$ gives the desired solution of the Dirichlet problem (5) in D for the Beltrami equation (1).

A wide range of effective criteria is well known for the existence of homeomorphic solutions to the degenerate Beltrami equation, defined in the whole complex plane, see e. g. historic comments with relevant references in monographs [3] and [9]—[11]. Assuming, for example, that the complex dilatation $\mu(z)$ has a compact support and the corresponding regular homeomorphic solution g to the Beltrami equation (1) is normalized by the condition $g(z) = z + o(1)$ at the infinity, we can apply to the study of the Dirichlet problem some criteria established in our last paper [12].

From now on, we assume that the functions $K_\mu^T(z, z_0)$ and $K_\mu(z)$ are extended by 1 outside of the domain D.

Proposition 2. *Let D be a bounded simply connected domain in $\mathbb{C}$. Suppose that $\mu : D \to \mathbb{C}$ is a measurable function with $|\mu(z)| < 1$ a. e., $K_\mu \in L^1(D)$ and*

$$\int\limits_{\varepsilon < |z - z_0| < \varepsilon_0} K_\mu^T(z, z_0) \cdot \psi_{z_0, \varepsilon}^2(|z - z_0|) \, dm(z) = o(I_{z_0}^2(\varepsilon)) \quad \text{as} \quad \varepsilon \to 0 \quad \forall \, z_0 \in \overline{D} \tag{6}$$

for some $\varepsilon_0 = \varepsilon(z_0) > 0$ and a family of measurable functions $\psi_{z_0, \varepsilon} : (0, \varepsilon_0) \to (0, \infty)$ with

$$I_{z_0}(\varepsilon) := \int\limits_\varepsilon^{\varepsilon_0} \psi_{z_0, \varepsilon}(t) \, dt < \infty \qquad \forall \varepsilon \in (0, \varepsilon_0). \tag{7}$$

Then the Beltrami equation (1) has a regular solution f of the Dirichlet problem (5) in D for each continuous function $\varphi : \partial D \to \mathbb{R}$, $\varphi(\zeta) \not\equiv \text{const}$.

Moreover, such a solution f can be represented as the composition

$$f = h \circ g, \qquad g(z) = z + o(1) \quad \text{as} \quad z \to \infty, \tag{8}$$

where $g : \mathbb{C} \to \mathbb{C}$ is a regular homeomorphic solution of the Beltrami equation (1) in $\mathbb{C}$ with μ extended by zero outside of D and $h : D_* \to \mathbb{C}, D_* := g(D)$, is a holomorphic solution of the Dirichlet problem

$$\lim_{\xi \to \zeta} \text{Re}\, h(\xi) = \varphi_*(\zeta) \quad \forall \, \zeta \in \partial D_*, \quad \text{where } \varphi_* := \varphi \circ g^{-1}. \tag{9}$$

Proof. By Lemma 1 in [12], there is a regular homeomorphic solution g to the Beltrami equation (1) in $\mathbb{C}$ with hydrodynamic normalization $g(z) := z + o(1)$ as $z \to \infty$. Consequently, by Proposition 1, the function $f := h \circ g$ gives the desired solution of the Dirichlet problem (5) in D for the Beltrami equation (1).

Remark 1. Note that if the family of the functions $\psi_{z_0, \varepsilon}(t) \equiv \psi_{z_0}(t)$ is independent on the parameter ε, then condition (6) implies that $I_{z_0}(\varepsilon) \to \infty$ as $\varepsilon \to 0$. This follows immediately from arguments by contradiction, apply for it (4) and the condition $K_\mu \in L^1(D)$. Note also that (6) holds, in particular, if, for some $\varepsilon_0 = \varepsilon(z_0)$,

$$\int\limits_{|z - z_0| < \varepsilon_0} K_\mu^T(z, z_0) \cdot \psi_{z_0}^2(|z - z_0|) \, dm(z) < \infty \qquad \forall \, z_0 \in \overline{D} \tag{10}$$

and $I_{z_0}(\varepsilon) \to \infty$ as $\varepsilon \to 0$. In other words, for the solvability of the Dirichlet problem (5) in D for the Beltrami equation (1) for all continuous boundary functions φ, it is sufficient that the integral in (10) converges for some nonnegative function $\psi_{z_0}(t)$ that is locally integrable over $(0, \varepsilon_0]$ but has a nonintegrable singularity at 0. The functions $\log^\lambda(e/|z - z_0|)$, $\lambda \in (0,1)$, $z \in \mathbb{D}$, $z_0 \in \overline{\mathbb{D}}$, and $\psi(t) = 1/(t \log(e/t))$, $t \in (0,1)$, show that the condition (10) is compatible with the condition $I_{z_0}(\varepsilon) \to \infty$ as $\varepsilon \to 0$. Furthermore, condition (6) shows that (10) is sufficient for the solvability of the Dirichlet problem, even if the integral in (10) is divergent in a controlled way.

2. Existence theorems. The definitions of classes BMO, which denotes functions of bounded mean oscillation, and FMO, which denotes functions of finite mean oscillation, can be found e. g., in the paper [12]. Choosing $\psi(t) = 1/(t \log(1/t))$ in Proposition 2 and applying Lemma 2 from [12], we obtain the following result.

Theorem 1. *Let D be a bounded simply connected domain in $\mathbb{C}$ and let a function $\mu : D \to \mathbb{C}$ be measurable with $|\mu(z)| < 1$ a. e. and $K_\mu \in L^1(D)$. Suppose also that $K_\mu^T(z, z_0) \leqslant Q_{z_0}(z)$ a. e. in U_{z_0} for every point $z_0 \in \overline{D}$, a neighborhood U_{z_0} of z_0 and a function $Q_{z_0} : U_{z_0} \to [0, \infty]$ in the class $FMO(z_0)$. Then the Beltrami equation (1) has a regular solution of the Dirichlet problem (5) in D with the representation (8) for each continuous function $\varphi : \partial D \to \mathbb{R}$, $\varphi(\zeta) \not\equiv \text{const}$.*

In particular, by Proposition 1 in [12] the conclusion of Theorem 1 holds if every point $z_0 \in \overline{D}$ is the Lebesgue point of the function Q_{z_0}.

Corollary 1. *Let D be a bounded simply connected domain in $\mathbb{C}$, $\mu : D \to \mathbb{C}$ be a measurable function with $|\mu(z)| < 1$ a.e. and K_μ have a majorant $Q : \mathbb{C} \to [1, \infty)$ in the class BMO. Then the Beltrami equation (1) has a regular solution of the Dirichlet problem (5) in D with the representation (8) for each continuous function $\varphi : \partial D \to \mathbb{R}$, $\varphi(\zeta) \not\equiv \text{const}$.*

By Corollary 2 in [12], we obtain the following nice consequence of Theorem 1.

Corollary 2. *Let D be a bounded simply connected domain in $\mathbb{C}$ and let $\mu : D \to \mathbb{C}$ be a measurable function with $|\mu(z)| < 1$ a. e., $K_\mu \in L^1(D)$ and*

$$\overline{\lim_{\varepsilon \to 0}} \frac{1}{\pi \varepsilon^2} \int_{B(z_0, \varepsilon)} K_\mu^T(z, z_0)\, dm(z) < \infty \qquad \forall z_0 \in \overline{D}. \tag{11}$$

Then the Beltrami equation (1) has a regular solution of the Dirichlet problem (5) in D with the representation (8) for each continuous function $\varphi : \partial D \to \mathbb{R}$, $\varphi(\zeta) \not\equiv \text{const}$.

Similarly, choosing in Proposition 2 $\psi(t) = 1/t$, we come to the next statement.

Theorem 2. *Let D be a bounded simply connected domain in $\mathbb{C}$ and let $\mu : D \to \mathbb{C}$ be a measurable function with $|\mu(z)| < 1$ a. e. and $K_\mu \in L^1(D)$. Suppose also that, for some $\varepsilon_0 = \varepsilon(z_0) > 0$, as $\varepsilon \to 0$*

$$\int_{\varepsilon < |z - z_0| < \varepsilon_0} K_\mu^T(z, z_0) \frac{dm(z)}{|z - z_0|^2} = o\left(\left[\log \frac{1}{\varepsilon}\right]^2\right) \qquad \forall z_0 \in \overline{D}. \tag{12}$$

Then the Beltrami equation (1) has a regular solution of the Dirichlet problem (5) in D with the representation (8) for each continuous function $\varphi : \partial D \to \mathbb{R}$, $\varphi(\zeta) \not\equiv \text{const}$.

Remark 2. Choosing in Lemma 1 the function $\psi(t) = 1/(t \log 1/t)$ instead of $\psi(t) = 1/t$, we can replace (12) by

$$\int\limits_{\varepsilon < |z - z_0| < \varepsilon_0} \frac{K_\mu^T(z, z_0) \, dm(z)}{\left(|z - z_0| \log \dfrac{1}{|z - z_0|} \right)^2} = o\left(\left[\log \log \frac{1}{\varepsilon} \right]^2 \right). \tag{13}$$

In general, we are able to give here the whole scale of the corresponding conditions using functions $\psi(t)$ of the form $1/(t \log 1/t \cdot \log \log 1/t \cdot \ldots \cdot \log \ldots \log 1/t)$.

Now, choosing in Proposition 2 $\psi_{z_0, \varepsilon}(t) \equiv \psi_{z_0}(t) = 1/[t k_\mu^T(z_0, t)]$, where $k_\mu^T(z_0, r)$ is the integral mean of $K_\mu^T(z, z_0)$ over circle $S(z_0, r) = \{ z \in \mathbb{C} : |z - z_0| = r \}$, we obtain one more important conclusion.

Theorem 3. *Let D be a bounded simply connected domain in $\mathbb{C}$ and let $\mu : D \to \mathbb{C}$ be a measurable function with $|\mu(z)| < 1$ a. e. and $K_\mu \in L^1(D)$. Suppose also that, for some $\varepsilon_0 = \varepsilon(z_0) > 0$,*

$$\int\limits_0^{\varepsilon_0} \frac{dr}{r k_\mu^T(z_0, r)} = \infty, \qquad \forall \, z_0 \in \overline{D}. \tag{14}$$

Then the Beltrami equation (1) has a regular solution of the Dirichlet problem (5) in D with the representation (8) for each continuous function $\varphi : \partial D \to \mathbb{R}$, $\varphi(\zeta) \not\equiv \text{const}$.

Corollary 3. *Let D be a bounded simply connected domain in $\mathbb{C}$ and let $\mu : D \to \mathbb{C}$ be a measurable function with $|\mu(z)| < 1$ a. e., $K_\mu \in L^1(D)$ and*

$$k_\mu^T(z_0, \varepsilon) = O\left(\log \frac{1}{\varepsilon} \right) \qquad \text{as } \varepsilon \to 0 \qquad \forall \, z_0 \in \overline{D}. \tag{15}$$

Then the Beltrami equation (1) has a regular solution of the Dirichlet problem (5) in D with the representation (8) for each continuous function $\varphi : \partial D \to \mathbb{R}$, $\varphi(\zeta) \not\equiv \text{const}$.

Remark 3. In particular, the conclusion of Corollary 3 holds if

$$K_\mu^T(z, z_0) = O\left(\log \frac{1}{|z - z_0|} \right) \qquad \text{as} \quad z \to z_0 \quad \forall \, z_0 \in \overline{D}. \tag{16}$$

The condition (15) can be also replaced by the whole series of weaker conditions

$$k_\mu^T(z_0, \varepsilon) = O\left(\left[\log \frac{1}{\varepsilon} \cdot \log \log \frac{1}{\varepsilon} \cdot \ldots \cdot \log \ldots \log \frac{1}{\varepsilon} \right] \right) \qquad \forall \, z_0 \in \overline{D}. \tag{17}$$

Combining Theorems 2.5 and 3.2 in [13] and Theorems 3, we obtain the following result.

Theorem 4. *Let D be a bounded simply connected domain in $\mathbb{C}$ and let $\mu : D \to \mathbb{C}$ be a measurable function with $|\mu(z)| < 1$ a .e. and $K_\mu \in L^1(D)$. Suppose also that, for a neighborhood U_{z_0} of z_0,*

$$\int\limits_{U_{z_0}} \Phi_{z_0}(K_\mu^T(z, z_0)) \, dm(z) < \infty \qquad \forall \, z_0 \in \overline{D}, \tag{18}$$

where $\Phi_{z_0} : [0, \infty] \to [0, \infty]$ *is a convex non-decreasing function such that*

$$\int\limits_{\Delta(z_0)}^{\infty} \log\Phi_{z_0}(t) \frac{dt}{t^2} = +\infty \tag{19}$$

for some $\Delta(z_0) > 0$. *Then Beltrami equation (1) has a regular solution of the Dirichlet problem (5) in* D *with the representation (8) for each continuous function* $\varphi : \partial D \to \mathbb{R}$, $\varphi(\zeta) \not\equiv \mathrm{const}$.

Corollary 4. *Let* D *be a bounded simply connected domain in* $\mathbb{C}$ *and let* $\mu : D \to \mathbb{C}$ *be a measurable function with* $|\mu(z)| < 1$ *a. e. and* $K_\mu \in L^1(D)$. *Suppose also that*

$$\int\limits_{D} \Phi(K_\mu(z)) \, dm(z) < \infty \tag{20}$$

for a convex non-decreasing function $\Phi : [0, \infty] \to [0, \infty]$ *such that, for* $\delta > 0$,

$$\int\limits_{\delta}^{\infty} \log \Phi(t) \frac{dt}{t^2} = +\infty. \tag{21}$$

Then the Beltrami equation (1) has a regular solution of the Dirichlet problem (5) in D *with the representation (8) for each continuous function* $\varphi : \partial D \to \mathbb{R}$, $\varphi(\zeta) \not\equiv \mathrm{const}$.

Remark 4. By the Stoilow theorem (see [14]), a regular solution f of the Dirichlet problem (5) in D for the Beltrami equation (1) with $K_\mu \in L^1_{loc}(D)$ can be represented in the form $f = h \circ F$ where h is a holomorphic function and F is a homeomorphic regular solution of (1) in the class $W^{1,1}_{loc}$. Thus, as shown in Theorem 5.1 of [13], condition (21) is not only sufficient but also necessary for the existence of a regular solution of the Dirichlet problem (5) in D for arbitrary Beltrami equations (1) with the integral constraints (20) for all continuous functions $\varphi : \partial D \to \mathbb{R}$, $\varphi(\zeta) \not\equiv \mathrm{const}$.

Corollary 5. *Let* D *be a bounded simply connected domain in* $\mathbb{C}$ *and let* $\mu : D \to \mathbb{C}$ *be a measurable function with* $|\mu(z)| < 1$ *a. e.,* $K_\mu \in L^1(D)$ *and*

$$\int\limits_{U_{z_0}} e^{\alpha(z_0)K_\mu^T(z, z_0)} dm(z) < \infty \qquad \forall\, z_0 \in \overline{D} \tag{22}$$

for some $\alpha(z_0) > 0$ *and a neighborhood* U_{z_0} *of the point* z_0. *Then the Beltrami equation (1) has a regular solution of the Dirichlet problem (5) in* D *with the representation (8) for each continuous function* $\varphi : \partial D \to \mathbb{R}$, $\varphi(\zeta) \not\equiv \mathrm{const}$.

Corollary 6. *Let* D *be a bounded simply connected domain in* $\mathbb{C}$ *and let* $\mu : D \to \mathbb{C}$ *be a measurable function with* $|\mu(z)| < 1$ *a. e. and, for some* $\alpha > 0$,

$$\int\limits_{D} e^{\alpha K_\mu(z)} dm(z) < \infty. \tag{23}$$

Then the Beltrami equation (1) has a regular solution of the Dirichlet problem (5) in D *with the representation (8) for each continuous function* $\varphi : \partial D \to \mathbb{R}$, $\varphi(\zeta) \not\equiv \mathrm{const}$.

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ПРО ЗАДАЧУ ДІРІХЛЕ ДЛЯ ВИРОДЖЕНОГО РІВНЯННЯ БЕЛЬТРАМІ

Вивчається задача Діріхле для виродженого рівняння Бельтрамі з неперервними межовими даними у довільній однозв'язній області комплексної площини. Встановлені критерії існування регулярних дискретних відкритих розв'язків цієї задачі, що відбулося шляхом використання функцій класів ВМО — обмеженого середнього коливання та FMO — скінченного середнього коливання, а також ряду ефективних інтегральних критеріїв. Більше того, нами показано, що вказані розв'язки можуть бути зображені у вигляді композиції регулярних гомеоморфних розв'язків рівнянь Бельтрамі з гідродинамічним нормуванням у нескінченно віддаленій точці та голоморфного розв'язку відповідної задачі Діріхле, яка є асоційованою з цим рівнянням. Головні критерії сформульовані в термінах дотичної і максимальної дилатацій. Отримані результати можуть бути застосовані для для механіки рідин в сильно анізотропних і неоднорідних середовищах, оскільки рівняння Бельтрамі є складною формою основного рівняння гідромеханіки.

Ключові слова: ВМО, обмежене середнє коливання, FMO, скінченне середнє коливання, задача Діріхле, вироджені рівняння Бельтрамі, однозв'язні області

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