On compact classes of solutions of Dirichlet problem in Jordan domains

Olexander Dovhopiatyi[1], Evgeny Sevost'yanov[2]

[1,2] *Zhytomyr Ivan Franko State University, University, Zhytomyr;*
[2] *Institute of Applied Mathematics and Mechanics of NAS of Ukraine, Slavyansk*

[1]Alexdov1111111@gmail.com, [2]esevostyanov2009@gmail.com

Consider the following Cauchy problem:

$$f_{\overline{z}} = \mu(z) \cdot f_z\,, \tag{1}$$

$$\lim_{\zeta \to z} \operatorname{Re} f(\zeta) = \varphi(z) \qquad \forall\ z \in \partial D\,, \tag{2}$$

where $\varphi : \partial D \to \mathbb{R}$ is a predefined continuous function. In what follows, we assume that D is some Jordan domain in $\mathbb{C}$.

Given $z_0 \in D$, a function $\varphi : \partial D \to \mathbb{R}$, a function $\Phi : \overline{\mathbb{R}^+} \to \overline{\mathbb{R}^+}$ and a function $\mathcal{M}(\Omega)$ of open sets $\Omega \subset D$, we denote by $\mathfrak{F}^{\mathcal{M}}_{\varphi, \Phi, z_0}(D)$ the class of all regular solutions $f : D \to \mathbb{C}$ of the Cauchy problem (1)–(2) that satisfy the condition $\operatorname{Im} f(z_0) = 0$ and, in addition,

$$\int\limits_{\Omega} \Phi(K_\mu(z)) \cdot \frac{dm(z)}{(1+|z|^2)^2} \leqslant \mathcal{M}(\Omega) \tag{3}$$

Theorem. *Let D be some bounded Jordan domain in $\mathbb{C}$, and let $\Phi : \overline{\mathbb{R}^+} \to \overline{\mathbb{R}^+}$ be a continuous increasing convex function, which satisfies the condition*

$$\int\limits_{\delta}^{\infty} \frac{d\tau}{\tau \Phi^{-1}(\tau)} = \infty$$

for some $\delta > \Phi(0)$. Assume that the function $\mathcal{M}$ is bounded, and the function φ in (2) is continuous. Then the family $\mathfrak{F}^{\mathcal{M}}_{\varphi, \Phi, z_0}(D)$ is compact in D.