On compact classes of solutions of Dirichlet problem in simply connected domains

Dovhopiatyi Oleksandr, Sevost'yanov Evgeny
Zhytomyr Ivan Franko State University

Let D be a domain in $\mathbb{C}$. In what follows, a mapping $f : D \to \mathbb{C}$ is assumed to be *sense-preserving,* moreover, we assume that f has partial derivatives almost everywhere. Put $f_{\overline{z}} = (f_x + if_y)/2$ and $f_z = (f_x - if_y)/2$. The *complex dilatation* of f at $z \in D$ is defined as follows: $\mu(z) = \mu_f(z) = f_{\overline{z}}/f_z$ for $f_z \neq 0$ and $\mu(z) = 0$ otherwise. The *maximal dilatation* of f at z is the following function: $K_\mu(z) = K_{\mu_f}(z) = \frac{1+|\mu(z)|}{1-|\mu(z)|}$.

Consider the following Cauchy problem:

$$f_{\overline{z}} = \mu(z) \cdot f_z\,,$$
$$\lim_{\zeta \to P} \operatorname{Re} f(\zeta) = \varphi(P) \qquad \forall\ P \in E_D\,,$$

where $\varphi : E_D \to \mathbb{R}$ is a predefined continuous function. In what follows, we assume that D is some simply connected domain in $\mathbb{C}$. The solution of this problem is called *regular,* if one of two conditions is fulfilled: or $f(z) = const$ in D, or f is an open discrete $W^{1,1}_{\rm loc}(D)$-mapping such that $J(z, f) \neq 0$ for almost any $z \in D$.

Given $z_0 \in D$, a function $\varphi : E_D \to \mathbb{R}$, a function $\Phi : \overline{\mathbb{R}^+} \to \overline{\mathbb{R}^+}$ and a function $\mathcal{M}(\Omega)$ of open sets $\Omega \subset D$, we denote by $\mathfrak{F}^{\mathcal{M}}_{\varphi,\Phi,z_0}(D)$ the class of all regular solutions $f : D \to \mathbb{C}$ of the Cauchy problem that satisfy the condition $\operatorname{Im} f(z_0) = 0$ and, in addition, $\int\limits_{\Omega} \Phi(K_\mu(z)) \cdot \frac{dm(z)}{(1+|z|^2)^2} \leqslant \mathcal{M}(\Omega)$

for any open set $\Omega \subset D$.

Theorem. *Let D be some simply connected domain in $\mathbb{C}$, and let $\Phi : \overline{\mathbb{R}^+} \to \overline{\mathbb{R}^+}$ be a continuous increasing convex function which satisfies the condition* $\int\limits_{\delta}^{\infty} \frac{d\tau}{\tau \Phi^{-1}(\tau)} = \infty$

for some $\delta > \Phi(0)$. Assume that the function $\mathcal{M}$ is bounded, and the function φ in Cauchy problem is continuous. Then the family $\mathfrak{F}^{\mathcal{M}}_{\varphi,\Phi,z_0}(D)$ is compact in D.

Monogenic functions and harmonic vectors

Sergiy Plaksa
Institute of Mathematics of the National Academy of Sciences of Ukraine

We consider special topological vector spaces with a commutative multiplication for some of elements of the spaces and monogenic functions taking values in these spaces. Monogenic functions are understood as continuous and differentiable in the sense of Gâteaux functions. We describe relations between the mentioned monogenic functions and harmonic vectors in the three-dimensional real space and establish sufficient conditions for infinite monogeneity of functions. Unlike the classical complex analysis, it is done in the case where the validity of the Cauchy integral formula for monogenic functions remains an open problem.

Biharmonic problem in an angle and monogenic functions

Serhii Gryshchuk, Sergiy Plaksa
Institute of Mathematics of NAS of Ukraine

We consider a piecewise continuous biharmonic problem in an angle and the corresponding Schwartz-type boundary-value problem for monogenic functions in a commutative biharmonic algebra. These problems are reduced to a system of integral equations on the positive semiaxis. It is