Oleg F. Gerus (Zhytomyr, Ukraine)

ON THE SPECTRUM OF OPERATORS CONCERNED WITH THE REDUCED SINGULAR CAUCHY INTEGRAL[1]

Let Γ be a rectifiable closed Jordan curve in the complex plane; $\Gamma_{z,\delta} := \{\zeta \in \Gamma : |\zeta - z| \leqslant \delta\}$, $\delta > 0$; $f : \Gamma \to \mathbb{C}$ be a continuous function;

$$\mathbf{F}[f](t) = \frac{1}{2\pi i}\int\limits_{\Gamma} \frac{f(\zeta)-f(t)}{\zeta - t}d\zeta := \frac{1}{2\pi i}\lim_{\delta\to 0}\int\limits_{\Gamma\setminus\Gamma_{t,\delta}} \frac{f(\zeta)-f(t)}{\zeta - t}d\zeta, \qquad t \in \Gamma,$$

be the reduced singular Cauchy integral.

The operator $\mathbf{F}$ is interpretable in the following shape:

$$\mathbf{F} = \mathbf{B} + i\mathbf{C},$$

where

$$\mathbf{B}[f](t) := -\frac{1}{2\pi}\int\limits_{\Gamma} \frac{f(\zeta)-f(t)}{|\zeta - t|^2}((\eta-\nu)d\xi - (\xi-\sigma)d\eta), \qquad t \in \Gamma,$$

$$\mathbf{C}[f](t) := -\frac{1}{2\pi}\int\limits_{\Gamma} \frac{f(\zeta)-f(t)}{|\zeta - t|^2}((\xi-\sigma)d\xi + (\eta-\nu)d\eta), \qquad t \in \Gamma,$$

and $\zeta := \xi + i\eta$, $t := \sigma + i\nu$.

We investigate spectrums of the operators $\mathbf{F}$, $\mathbf{B}$ and $\mathbf{C}$. Among others we proved that numbers 0 and -1 are proper numbers of infinite order of the operator $\mathbf{F}$. The point spectrum of the operator $\mathbf{B}$ excepting points 0 and -1 is reflected in the point $-\frac{1}{2}$. And the point spectrum of the operator $\mathbf{C}$ is reflected in the origin.

There is a close connection between spectrums of operators $\mathbf{B}$ and $\mathbf{C}$, owing to the equalities

$$\begin{cases} \mathbf{C}^2 - \mathbf{B}^2 = \mathbf{B}, \\ \mathbf{BC} + \mathbf{CB} = -\mathbf{C}. \end{cases}$$

So there exists a one-to-one correspondence between the eigenvalues $\lambda \in (-1, -\frac{1}{2}) \cup (-\frac{1}{2}, 0)$ of the operator $\mathbf{B}$ and $\gamma = i\sigma$, $\sigma \in (-\frac{1}{2}, 0) \cup (0, \frac{1}{2})$, of the operator $\mathbf{C}$ by the formula $\sigma^2 = -\lambda^2 - \lambda$.

[1]Partially supported by Fundamental Researches State Fund, project 25.1/084.