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APPLICATION OF SYMMETRIC POLYNOMIALS

...“To be beautiful means to be symmetric and proportional”
Platon

People use a lot of approaches, ideas, methods in order to know the world. The most fundamental among them is the idea of symmetry. It is difficult to find a person who does not have any idea about symmetry. “Symmetry” is a word of the Greek origin. It, as well as the word “harmony”, means correspondence, presence of some order, peculiarities of the parts arrangement.

The purpose of work is a search of ways to solve various problems of algebra using symmetry.

Symmetry is widely used in algebra. An example of symmetry application may be found in symmetric polynomials.

A polynomial $f(x_1, x_2, ..., x_n)$ is called symmetric regard to variables $(x_1, x_2, ..., x_n)$, if, as a result of arbitrary transposition of variables $(x_1, x_2, ..., x_n)$, we will get a polynomial that equals a given one.

A polynomial $x^2y + xy^2$ is symmetric. Opposite, a polynomial $x^3 - 3y^3$ is nonsymmetric: substituting *x* by *y*, and *y* by *x* it transforms into a polynomial $y^3 - 3x^3$ that does not coincide with the primary one.

We have already met with the important examples of symmetric polynomials while studying the theorem of Viet [2, c. 298].

If we mark $x_1, x_2, ..., x_n$, as the roots of polynomial $f(x) = x^n + a_{n-1}x^{n-1} + ... + a_1x + a_0$ then the Viet’s formulas will be represented as follows:

$$x_1 + x_2 + ... + x_n = -a_{n-1}$$

$$x_1x_2 + x_1x_3 + ... + x_{n-1}x_n = a_{n-2}$$

.....

$$x_1x_2....x_n = (-1)^n a_0$$

Signifying the left parts of these formulas through $\sigma_1, \sigma_2, ..., \sigma_n$ then we will get:

$$\sigma_1 = x_1 + x_2 + ... + x_n,$$

$$\sigma_2 = x_1x_2 + x_1x_3 + ... + x_{n-1}x_n$$

.....

$$\sigma_n = x_1x_2...x_n$$

According to these operations we get basic symmetric polynomials.

There are a lot of tasks, where it is necessary to find some expressions that contain the radicals of a quadratic equation. It is possible to solve these tasks using symmetric polynomials.

Example. Let’s take a quadratic equation $x^2 + 6x + 10 = 0$. Your task is to make a new equation the radicals of which are roots of this equation.

We will denote the radicals of the set equation by x_1 and x_2, the radicals of the desired equation by y_1 and y_2, coefficients of the desired equation by *p* and *q*. According to the Viet’s theorem for this equation we have the following: $\sigma_1 = x_1 + x_2 = -6$
$\sigma_2 = x_1x_2 = 10$

The same is for the desired equation: $y_1 + y_2 = -p$
$y_1y_2 = q$

By the data of the problem we have: $y_1 = x_1^2$, $y_2 = x_2^2$. This allows to write the formula for the coefficients: $p = -(y_1 + y_2) = -(x_1^2 + x_2^2) = -S_2$,

$q = y_1 y_2 = x_1^2 x_2^2 = \sigma_2^2 = 100$

where S_2 is the second sum of powers, which is expressed through basic symmetric polynomials using the sum of powers table [1, c.47].

$$p = -(\sigma_1^2 - 2\sigma_2) = -16$$
$$q = \sigma_2^2 = 100$$

So, the desired quadratic equation has the form $y^2 - 16y + 100 = 0$.

Taking everything into account, symmetry takes an important place in algebra particularly in application of symmetric polynomials.

LITERATURE

1. Болтянский В. Г., Виленкин Н. Я. Симметрия в алгебре. – М.: МЦНМО, 2002. – 240 с.
2. Завало С. Т., Костарчук В. Н., Хацет Б. І. Алгебра і теорія чисел. Частина 2. – К.: Вища шк., 1980. – 406 с.