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MATH PROBLEMS WITH FRACTALS

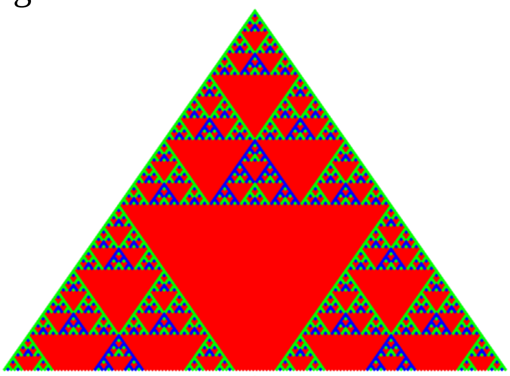
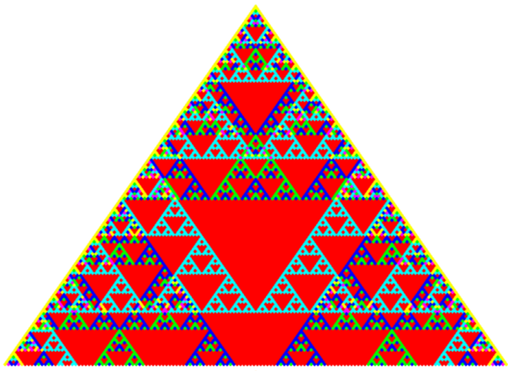
When we want to describe man-made objects such as a table, a tennis ball or an Egyptian pyramid, we use Euclidean geometry. But it's impossible to describe such natural objects as mountains, clouds or coastlines using only Euclidean geometry because their shape is much more complicated than a set of simple geometric figures. In 1975 the French mathematician Benoit Mandelbrot introduced a new mathematical concept "fractal", which solved the problem of describing complicated natural objects and significantly changed the vision of modern mathematics. Mandelbrot wrote in his work "The Fractal Geometry of Nature" that we didn't have to ask ourselves why we needed fractals, it would be better to ask why we didn't think of them earlier [3].

Fractal (from Latin *fractus*, "crushed, fractional") is an irregular self-similar structure, small parts of which are similar to the whole figure [1]. One of the methods of creating fractals is the method of simple replacement, when one element is replaced by several others, and such process can be repeated an infinite number of times. One such replacement step is called an iteration (from Latin *iteratio*, "repeat") [2].

The examples of natural fractals are a branch of fern, Romanesco broccoli, a coastline, bronchi, brain, some forms of snowflakes, etc. Fractals are used in physics, electrical engineering, medicine, biology, mathematics, astronomy, psychology, economics, art and other sciences. This fact proves that fractals occupy an important place in our lives.

This article focuses on the math problems connected with fractals. We have created a set of math problems, which can be divided into two groups: 1) the problems with a finite number of iterations; 2) the problems with an infinite number of iterations. The example below illustrates the solution of the math problem of the second group containing a geometrical fractal, in particular, the Sierpinski triangle.

Problem: Find the area of the Sierpinski triangle obtained from the equilateral triangle with the area 1.

Solution: To find the area of the Sierpinski triangle, we need to find the area of all triangles that we've cut out. At first we cut out one triangle, the area of which is ¼ of the whole triangle. With each new stage the number of triangles becomes 3 times more and the area of each triangle becomes 4 times less. According to the formula of the sum of the infinite geometric progression ($S=\frac{b_1}{1-q}$, $|q|<1$, where b_1 is the first term of the geometric progression, and q is the geometric ratio of the progression; $b_1=\frac{1}{4}, q=\frac{3}{4}$) we find out that the sum of the areas of all the triangles that were cut out, is equal to 1. Therefore, the area of the Sierpinski triangle is equal to 0.

Math problems with fractals can be used during the lessons of Math and optional courses at schools. In our opinion, solving such problems will familiarize pupils with the world of fractals, increase their interest in Math and contribute to the development of their logical thinking, spatial imagination and creative abilities.

LITERATURE

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