On mappings with inverse Poletsky inequality on Riemannian manifolds

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Let $\mathbb{M}^n$ and $\mathbb{M}^n_*$ are Riemannian manifolds of dimension n with geodesic distances d and d_*, respectively, $dv(x)$ and $dv_*(x)$ are volume measures on $\mathbb{M}^n$ and $\mathbb{M}^n_*$, respectively. Let $x_0 \in D$, and the number $r_0 > 0$ be such that the ball $B(x_0, r_0)$ lies in some normal neighborhood U of the point x_0 with its closure. Given sets E, F and G in $\mathbb{M}^n$, we denote by $\Gamma(E, F, G)$ the family of all paths $\gamma\colon [a, b] \to \mathbb{M}^n$, joining E and F in G. Denote $S_i = S(x_0, r_i)$, $i = 1, 2$, geodesic spheres centered at the point x_0 and radii r_1 and r_2. If $y_0 \in f(D)$ and $0 < r_1 < r_2 < d_0 = \sup\limits_{y \in f(D)} d_*(y, y_0)$, we denote by $\Gamma_f(y_0, r_1, r_2)$ the family of all paths γ in the domain D such that $f(\gamma) \in \Gamma(S(y_0, r_1), S(y_0, r_2), A(y_0, r_1, r_2))$. Let $Q : \mathbb{M}^n_* \to [0, \infty]$ be a measurable function with respect to the volume measure v_*, and let $M(\cdot)$ be a modulus of families of paths. We say that f *satisfies the inverse Poletskii inequality* at the point $y_0 \in f(D)$, if the relation $M(\Gamma_f(y_0, r_1, r_2)) \leqslant \int\limits_{A(y_0,r_1,r_2)\cap f(D)} Q(y) \cdot \eta^n(d_*(y, y_0))\, dv_*(y)$ holds for any Lebesgue measurable function $\eta : (r_1, r_2) \to [0, \infty]$ such that $\int\limits_{r_1}^{r_2} \eta(r)\, dr \geqslant 1$. In what follows, $q_{x_0}(r) = \frac{1}{r^{n-1}} \int\limits_{S(x_0,r)} Q(x)\, d\mathcal{A}$, where $d\mathcal{A}$ is the area element of $S(x_0, r)$.

For domains $D \subset \mathbb{M}^n$, $D_* \subset \mathbb{M}^n_*$, $n \geqslant 2$, and a function $Q\colon \mathbb{M}^n_* \to [0, \infty]$, $Q(x) \equiv 0$ for $x \notin D_*$, denote by $\mathfrak{R}_Q(D, D_*)$ the family of all open discrete mappings $f\colon D \to \mathbb{M}^n_*$, $f(D) = D_*$, for which f satisfies the inverse Poletsky inequality at each point $y_0 \in D_*$. The following result holds.

Theorem. *Assume that, $\overline{D}$ and $\overline{D_*}$ a compact sets in $\mathbb{M}^n$ and $\mathbb{M}^n_*$, respectively, $\overline{D}_* \neq \mathbb{M}^n_*$ and, in addition, $\mathbb{M}^n_*$ is connected. Suppose also that the following condition is satisfied: for each point $y_0 \in \overline{D_*}$ there is $r_0 = r_0(y_0) > 0$ such that $q_{y_0}(r) < \infty$ for each $r \in (0, r_0)$. Then the family $\mathfrak{R}_Q(D, D_*)$ is equicontinuous in D.*