

## On equicontinuity of some class of mappings, which are quasiconformal in the mean

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Given  $E$  and  $F \subset \overline{\mathbb{R}^n}$ , denote  $\Gamma(E, F, D)$  the family of all curves  $\gamma : [a, b] \rightarrow \overline{\mathbb{R}^n}$  joining  $E$  and  $F$  in  $D$ . In what follows,  $h$  is a chordal metric. Let  $Q(x, t) = \{y \in \overline{\mathbb{R}^n} : h(x, y) < t\}$  be the spherical ball centered at  $x$  of radius  $t$ . Let  $c(E, x) = \max\{m(E, x), m(E, \tilde{x})\}$ , where  $c(E) = \inf_{x \in \overline{\mathbb{R}^n}} c(E, x)$ ,  $m_t(E, r, x) = M(\Gamma(\partial Q(x, t), E \cap \overline{Q(x, r)}))$ ,  $m(E, x) = m_{\sqrt{3}/2}(E, \frac{\sqrt{2}}{2}, x)$ . Let  $\Phi : [0, \infty] \rightarrow [0, \infty]$  be a non-decreasing convex function. Denote  $\mathfrak{R}_{M, \Delta}^\Phi$  the family of all open discrete ring  $Q$ -mappings in  $D$  with  $c(\overline{\mathbb{R}^n} \setminus f(D)) \geq \Delta$ , and

$$\int_D \Phi(Q(x)) \frac{dm(x)}{(1 + |x|^2)^n} \leq M.$$

**Theorem.** *Let  $\Phi : [0, \infty] \rightarrow [0, \infty]$  be a non-decreasing convex function. If*

$$\int_{\delta_0}^{\infty} \frac{d\tau}{\tau [\Phi^{-1}(\tau)]^{\frac{1}{n-1}}} = \infty$$

for some  $\delta_0 > \tau_0 := \Phi(0)$ , then  $\mathfrak{R}_{M,\Delta}^\Phi$  is equicontinuous, and, consequently, forms a normal family of mappings for all  $M \in (0, \infty)$  and  $\Delta \in (0, 1)$ .

Let  $\mathfrak{R}_{Q,\Delta}(D)$  be a class of all open discrete ring  $Q$ -mappings  $f$  in a domain  $D \subset \mathbb{R}^n$ ,  $n \geq 2$ , with  $c(\overline{\mathbb{R}^n} \setminus f(D)) \geq \Delta > 0$ . To prove the theorem mentioned above, we have established the following assertion.

**Lemma.** Let  $\Delta > 0$  and let  $Q : D \rightarrow [0, \infty]$  be a measurable function. Now

$$h(f(x), f(x_0)) \leq \frac{\omega_{n-1}}{c_n \Delta} \cdot \left( \int_{|x-x_0|}^{\varepsilon(x_0)} \frac{dr}{r q_{x_0}^{\frac{1}{n-1}}(r)} \right)^{-1}$$

for every  $f \in \mathfrak{R}_{Q,\Delta}(D)$  and  $x \in B(x_0, \varepsilon(x_0))$ ,  $\varepsilon(x_0) < \text{dist}(x_0, \partial D)$ , where  $c_n > 0$  depends only on  $n$ , and  $q_{x_0}(r)$  is a mean integral value of the function  $Q(z)$  on  $|z - x_0| = r$ .

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