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ON BOUNDARY DISTORTION ESTIMATES OF MAPPINGS IN DOMAINS WITH POINCARE INEQUALITY

Let $Q : \mathbb{R}^n \to [0, \infty]$ be a Lebesgue measurable function that vanishes outside D. In view of [section 7.6,1], we will say that the mapping $f : D \to \overline{\mathbb{R}^n}$ is a *ring Q-mapping at the point $x_0 \in \overline{D}$ with respect to p-module,* $x_0 \neq \infty$, $p \geqslant 1$, if there is $r_0 = r(x_0) > 0$ such that for arbitrary $0 < r_1 < r_2 < r_0$ the following inequality holds

$$M_p(f(\Gamma(S(x_0, r_1),\ S(x_0, r_2),\ D))) \leqslant \int\limits_A Q(x) \cdot \eta^p(|x - x_0|)\ dm(x)\,, \qquad (1)$$

where $\eta : (r_1, r_2) \to [0, \infty]$ is an arbitrary non-negative Lebesgue-dimensional function such that

$$\int\limits_{r_1}^{r_2} \eta(r)\, dr \geqslant 1\,. \qquad (2)$$

Given [section 7.22,2], we say that the borel function $\rho\colon D \to [0, \infty]$ is *the upper gradient* for $u\colon D \to \mathbb{R}$, if the inequality $|u(x) - u(y)| \leqslant \int\limits_\gamma \rho\,|dx|$ is satisfied for all the smooth curves γ, connecting the points x and $y \in D$. We say that in the domain D the Poincare inequality $(1; p)$, $p \geqslant 1$, holds if there exist constants $C \geqslant 1$ and $\tau > 0$ such that for each ball $B \subset D$, of an arbitrary bounded continuous function $u\colon D \to \mathbb{R}$ and each of its upper gradients ρ holds

$$\frac{1}{m(B)}\int\limits_B |u(x)-u_B|\,dm(x) \leqslant C\cdot(\operatorname{diam} B)\left(\frac{1}{m(\tau B)}\int\limits_{\tau B}\rho^p(x)\,dm(x)\right)^{1/p},$$

where $u_B := \frac{1}{m(B)}\int\limits_B u(x)\,dm(x)$. A domain D is called *Ahlfors regular,* if there exists a constant $C \geqslant 1$ such that for every $x_0 \in D$ and any $R < \operatorname{diam} D$ the inequalities $\frac{1}{C}R^n \leqslant m(B(x_0,R)\cap D) \leqslant CR^n$ holds. Let $A, B \subset \mathbb{R}^n$. Let's put $\operatorname{diam} A = \sup\limits_{x,y\in A}|x-y|$, $\operatorname{dist}(A,B) = \inf\limits_{x\in A, y\in B}|x-y|$.

For $\delta > 0$ and $p \geqslant 1$, the domains $D, D' \subset \mathbb{R}^n$, $n \geqslant 2$, $x_0 \in \partial D$, continuum $A \subset D$ and the Lebesgue measurable function $Q : D \to [0,\infty]$ denoted by $\mathfrak{F}^{p,x_0}_{Q,A,\delta}(D,D')$ the family of all ring Q-homeomorphisms f of D onto D' at the point x_0 with respect to the p-module, satisfying the condition $\operatorname{diam}(f(A)) \geqslant \delta$. The following result is obtained.

Theorem 1. *Let $x_0 \in \partial D$, $x_0 \neq \infty$, $n-1 < p \leqslant n$. Assume that D' is a regular Alfors bounded domain with $(1;p)$-Poincare inequality, and the following conditions are satisfied: 1) there exists $r_0' = r_0'(x_0) > 0$ such that the set $B(x_0,r)\cap D$ is connected for all $0 < r < r_0'$; 2) there exists $\delta_0 = \delta_0(x_0) > 0$ such that.*

$$\int\limits_{\varepsilon}^{\delta_0}\frac{dt}{t^{\frac{n-1}{p-1}}q_{x_0}^{\frac{1}{p-1}}(t)} < \infty \qquad \forall\quad \varepsilon\in(0,\delta_0), \qquad \int\limits_{0}^{\delta_0}\frac{dt}{t^{\frac{n-1}{p-1}}q_{x_0}^{\frac{1}{p-1}}(t)} = \infty. \tag{3}$$

Then every $f \in \mathfrak{F}^{p,x_0}_{Q,A,\delta}(D,D')$ has a continuous extension to the point x_0, in addition, there are $\varepsilon_0 > 0$ and $\widetilde{C} > 0$, such that for all $x, y \in B(x_0,\varepsilon_0)\cap D$, $|x-x_0| \geqslant |y-x_0|$, and for all $f \in \mathfrak{F}^{p,x_0}_{Q,A,\delta}(D,D')$ is valid next evaluation

$$|f(x)-f(y)| \leqslant \widetilde{C}\cdot\left(\int\limits_{|x-x_0|}^{\varepsilon_0}\frac{dt}{t^{\frac{n-1}{p-1}}q_{x_0}^{\frac{1}{p-1}}(t)}\right)^{1-p}. \tag{4}$$

R E F E R E N C E S

1. *Martio O., Ryazanov V., Srebro U. and Yakubov E.* Moduli in Modern Mapping Theory. – New York: Springer Science + Business Media, LLC, 2009.

2. *Heinonen J.* Lectures on Analysis on metric spaces. New York: Springer Science+Business Media, 2001.