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ON THE THEORY OF GENERALIZED QUASICONFORMAL MAPPINGS

Let us give the basic definitions. Let Γ be a family of paths γ in $\mathbb{R}^n$. A Borel function $\rho:\mathbb{R}^n\rightarrow[0,\infty]$ is called *admissible* for Γ if $\int\limits_{\gamma}\rho(x)|dx|\geqslant 1$ for all (locally rectifiable) paths $\gamma\in\Gamma$. In this case, we write: $\rho\in\operatorname{adm}\Gamma$. Given a number $q\geqslant 1$, *q-modulus* of the family of paths Γ is defined as $M_q(\Gamma)=\inf\limits_{\rho\in\operatorname{adm}\Gamma}\int\limits_{D}\rho^q(x)\,dm(x)$. Let $x_0\in\overline{D}$, $x_0\neq\infty$, then

$$B(x_0,r)=\{x\in\mathbb{R}^n: |x-x_0|<r\}\,,$$

$$A=A(x_0,r_1,r_2)=\{x\,\in\,\mathbb{R}^n: r_1<|x-x_0|<r_2\}\,.$$

Given sets E, $F\subset\overline{\mathbb{R}^n}$ and a domain $D\subset\mathbb{R}^n$, we denote $\Gamma(E,F,D)$ a family of all paths $\gamma:[a,b]\rightarrow\overline{\mathbb{R}^n}$ such that $\gamma(a)\in E,\gamma(b)\in\,F$ and $\gamma(t)\in D$ for all $t\in(a,b)$.

Let $Q:\mathbb{R}^n\rightarrow[0,\infty]$ be a Lebesgue measurable function. We say that f *satisfies the Poletsky inverse inequality with respect to q-modulus* at a point $y_0\in f(D)$, $1<q<\infty$, if the moduli inequality

$$M_q(\Gamma(E,F,D))\leqslant\int\limits_{A(y_0,r_1,r_2)\cap f(D)}Q(y)\cdot\eta^q(|y-y_0|)\,dm(y)\qquad(1)$$

holds for any continua $E\subset f^{-1}(\overline{B(y_0,r_1)})$, $F\subset f^{-1}(f(D)\setminus B(y_0,r_2))$, $0<r_1<r_2<r_0=\sup\limits_{y\in f(D)}|y-y_0|$, and any Lebesgue measurable function $\eta:(r_1,r_2)\rightarrow[0,\infty]$ such that

$$\int\limits_{r_1}^{r_2}\eta(r)\,dr\geqslant 1\,.$$

Let D, D' be domains in $\mathbb{R}^n$, $n \geqslant 2$. For numbers $1 \leqslant q < \infty$ and a Lebesgue measurable function $Q : \mathbb{R}^n \to [0, \infty]$, $Q = 0$ a.e. on $\mathbb{R}^n \setminus D'$, we denote be $\mathfrak{R}^q_Q(D, D')$ the family of all open and discrete mappings $f : D \to D'$ such that the moduli inequality (1) holds at any point $y_0 \in D'$.

Theorem. *Let $Q \in L^1(\mathbb{R}^n)$ and $q \geqslant n$. Suppose that, K is compact in D, and D' is bounded. Then there exists a constant $C = C(n, q, K, \|Q\|_1, D, D') > 0$ such that the inequality*

$$|f(x) - f(y)| \leqslant C_n \cdot \frac{(\|Q\|_1)^{\frac{1}{q}}}{\log^{\frac{1}{n}}\left(1 + \frac{r_0}{2|x-y|}\right)}\,, \ r_0 = d(K, \partial D),$$

holds for any $x, y \in K$ and $f \in \mathfrak{R}_Q(D, D')$, where $\|Q\|_1$ denotes the L^1-norm of the function Q in $\mathbb{R}^n$.