Evgeny Sevost'yanov
Zarina Kovba
Heorhii Nosal
Nataliya Ilkevych
(Zhytomyr, Ukraine)

ON ONE ANALOG OF NÄKKI THEOREM FOR NON-CONFORMAL MODULI

Let us give some definitions. A Borel function $\rho : \mathbb{R}^n \to [0, \infty]$ is called *admissible* for family Γ of paths γ in $\mathbb{R}^n$, if $\int\limits_{\gamma} \rho(x)\,|dx| \geqslant 1$ holds for any (locally rectifiable) path $\gamma \in \Gamma$. In this case, we write: $\rho \in \operatorname{adm}\Gamma$. Given $p \geqslant 1$, we define *p-modulus* of the family Γ by the equality $M_p(\Gamma) = \inf\limits_{\rho \in \operatorname{adm}\Gamma} \int\limits_{\mathbb{R}^n} \rho^p(x)\,dm(x)$. Set $M(\Gamma) := M_n(\Gamma)$. Let $E_0, E_1 \subset \overline{\mathbb{R}^n}$, $\overline{\mathbb{R}^n} = \mathbb{R}^n \cup \{\infty\}$, and let D be a domain in $\overline{\mathbb{R}^n}$, $n \geqslant 2$. Set $M(\Gamma) = M_n(\Gamma)$. Denote by $\Gamma(E_0, E_1, D)$ the family of all paths joining E_0 and E_1 in D. The following result was established by R. Näkki (see [1, Lemma 1.15]).

Theorem A. (The positivity of the modulus of families of paths joining a pair of continua). *Let D be a domain in $\mathbb{R}^n$, $n \geqslant 2$. If A and A^* are (nondegenerate) continua in D, then $M(\Gamma(A, A^*, D)) > 0$.*

We have proved Näkki's result to the case of a modulus of order $p \geqslant 1$. The following theorem holds.

Theorem A_1. *Let D be a domain in $\mathbb{R}^n$, $n \geqslant 2$, and let $p > n - 1$. If A and A^* are (nondegenerate) continua in D, then $M_p(\Gamma(A, A^*, D)) > 0$.*

Let $p \geqslant 1$ and let D be a domain in $\mathbb{R}^n$, $n \geqslant 2$. We say that ∂D is *strongly accessible at the point $x_0 \in \partial D$ with respect to p-modulus,* if for any neighborhood U of x_0 there is a compact set $E \subset D$, a neighborhood $V \subset U$ of x_0 and a number $\delta > 0$ such that $M_p(\Gamma(E, F, D)) \geqslant \delta$ for each continuum F in D with $F \cap \partial U \neq \varnothing \neq F \cap \partial V$.

Theorem B. *Let D be a domain in $\mathbb{R}^n$, $n \geqslant 2$, which has strongly accessible boundary at the point $x_0 \in \partial D \setminus \{\infty\}$ with respect to p-modulus, $p > n - 1$. Then D is finitely connected at the point x_0, in other words, for any neighborhood U of x_0 there exists a neighborhood $V \subset U$ of the same point such that $V \cap D$ has a finite number of components.*

R E F E R E N C E S

1. Näkki, R. *Boundary behavior of quasiconformal mappings in n-space*, Ann. Acad. Sci. Fenn. Ser. A. **484**, 1–50 (1970).