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ON EXISTENCE OF SOLUTIONS OF BELTRAMI EQUATIONS IN THE CONTEXT OF TANGENTIAL DILATATION

Let $\mu = \mu(z, w) : D \times \mathbb{C} \to \mathbb{D}$ be $\nu = \nu(z, w) : D \times \mathbb{C} \to \mathbb{D}$ some functions. Let us consider the equation

$$f_{\overline{z}} = \mu(z, f(z)) \cdot f_z + \nu(z, f(z)) \cdot \overline{f_z}\,, \qquad (1)$$

where $f_{\overline{z}} = (f_x + if_y)/2$ and $f_z = (f_x - if_y)/2$. Fix $n \geqslant 1$ and set

$$\mu_n(z, w) = \begin{cases} \mu(z, w), & K_{\mu,\nu} \leqslant n, \\ 0\,, & K_{\mu,\nu} > n\,, \end{cases}$$

and

$$\nu_n(z, w) = \begin{cases} \nu(z, w), & K_{\mu,\nu} \leqslant n, \\ 0\,, & K_{\mu,\nu} > n\,, \end{cases}$$

where $K_{\mu,\nu}(z, w) = \frac{1+|\mu(z,w)|+|\nu(z,w)|}{1-|\mu\,(z,w)|-|\nu(z,w)|}$. Assume that, $\mu = \mu(z, w) : D \times \mathbb{C} \to \mathbb{D}$ and $\nu = \nu(z, w) : D \times \mathbb{C} \to \mathbb{D}$ satisfy Caratheodory conditions, i.e., ν is measurable by $z \in D$ for all fixed $w \in \mathbb{C}$, and continuous by w for almost all $z \in D$. Now

$$K_{\mu_n,\nu_n}(z, w) = \frac{1 + |\mu_n(z, w)| + |\nu_n(z, w)|}{1 - |\mu_n\,(z, w)| - |\nu_n(z, w)|} \leqslant n$$

for almost all $z \in D$ and all $w \in \mathbb{C}$. Now the equation $f_{\overline{z}} = \mu_n(z, f(z)) \cdot f_z + \nu_n(z, f(z)) \cdot \overline{f_z}$ has a homeomorphic solution f_n in $W^{1,1}_{\rm loc}(D)$ such that $f_n^{-1} \in W^{1,2}_{\rm loc}(f_n(D))$ and $f_n(0) = 0$, $f_n(1) = 1$.

Let f_n be a solution of (1) and $g_n = f_n^{-1}$. Set $K^T_{\mu_{g_n}}(w,w_0) = \frac{\left|1-\frac{\overline{w-w_0}}{w-w_0}\mu_{g_n}(w)\right|^2}{1-|\mu_{g_n}(w)|^2}$ and $K_{I,p}(w,g_n) = \frac{|(g_n)_w|^2-|(g_n)_{\overline{w}}|^2}{(|(g_n)_w|-|(g_n)_{\overline{w}}|)^p}$.

Theorem. *Let μ, ν, μ_n, ν_n, f_n and g_n be defined as above. Let $Q, Q_0 : \mathbb{C} \to [0,\infty]$ ne Lebesgue measurable functions. Assume that $K_{\mu,\nu}(z,w) \leqslant Q_0(z) < \infty$ for all $w \in \mathbb{C}$ and almost all $z \in D$. Assume that the following conditions hold:*

1) for all $0 < r_1 < r_2 < 1$ and $y_0 \in \mathbb{C}$ there is $E \subset [r_1, r_2]$ of a positive Lebesgue measure such that Q is integrable over $S(y_0, r)$ for all $r \in E$;

2) there are $1 < p \leqslant 2$ and $M > 0$ such that $\int\limits_{f_n(D)} K_{I,p}(w,g_n)\,dm(w) \leqslant M$ for all $n = 1, 2, \ldots$, where $K_{I,p}(w,g_n)$ is defined as above;

3) the relation

$$K^T_{\mu_{g_n}}(w,w_0) \leqslant Q(w)$$

holds for all $w \in f_n(D)$ and all $w_0 \in f_n(D)$, where $K^T_{\mu_{g_n}}$ is defined above. Then the equation (1) has a continuous $W^{1,p}_{\rm loc}(D)$-solution f in D.