Nataliya Ilkevych
Denys Romash
Evgeny Sevost'yanov
(Zhytomyr, Ukraine)

ON CONVERGENCE OF SOME CLASS OF MAPPINGS IN METRIC SPACES

Everywhere further, (X, d, μ) and (X', d', μ') are metric spaces with metrics d and d' and locally finite Borel measures μ and μ', correspondingly. Let G and G' be domains with finite Hausdorff dimensions α and $\alpha' \geqslant 2$ in (X, d, μ) and (X', d', μ'), respectively. For $x_0 \in X$ and $r > 0$, $S(x_0, r)$ denotes the sphere $\{x \in X : d(x, x_0) = r\}$. Put $d(E) := \sup\limits_{x,y\in E} d(x, y)$. Given $0 < r_1 < r_2 < \infty$, denote $A = A(x_0, r_1, r_2) = \{x \in X : r_1 < d(x, x_0) < r_2\}$. Let $p \geqslant 1$ and $q \geqslant 1$, and let $Q : G \to [0, \infty]$ be a measurable function. Due to [1], a homeomorphism $f : G \to G'$ is called a *ring Q-homeomorphism at a point $x_0 \in \overline{G}$ with respect to* (p, q)*-moduli*, if the inequality

$$M_p(f(\Gamma(S(x_0, r_1), S(x_0, r_2), A(x_0, r_1, r_2))))$$
$$\leqslant \int\limits_{A(x_0,r_1,r_2)\cap G} Q(x) \cdot \eta^q(d(x, x_0))\, d\mu(x)$$

holds for all $0 < r_1 < r_2 < r_0 := d(G)$ and each measurable function $\eta : (r_1, r_2) \to [0, \infty]$ with $\int\limits_{r_1}^{r_2} \eta(r)\, dr \geqslant 1$. We say that $f : G \to G'$ is a *ring Q-homeomorphism at a point $x_0 \in \overline{G}$, if the latter is true for $p = \alpha'$ and $q = \alpha$.*

We say that the condition of the *complete divergence of paths* is satisfied in $D \subset X$ if for any different points y_1 and $y_2 \in D$ there are some w_1,

$w_2 \in \partial D$ and paths $\alpha_2 : (-2,-1] \to D$, $\alpha_1 : [1,2) \to D$ such that 1) α_1 and α_2 are subpaths of some geodesic path $\alpha : [-2,2] \to X$, that is, $\alpha_2 := \alpha|_{(-2,-1]}$ and $\alpha_1 := \alpha|_{[1,2)}$; 2) the geodesic path α joins the points w_2, y_2, y_1 and w_1 such that $\alpha(-2) = w_2$, $\alpha(-1) = y_2$, $\alpha(1) = y_1$, $\alpha(2) = w_1$.

Let $\dim X = n$. For each real number $n \geqslant 1$, we define *the Loewner function* $\Phi_n : (0,\infty) \to [0,\infty)$ on X as $\Phi_n(t) = \inf\{M_n(\Gamma(E,F,X)) : \Delta(E,F) \leqslant t\}$, where the infimum is taken over all disjoint nondegenerate continua E and F in X and

$$\Delta(E,F) := \frac{\mathrm{dist}\,(E,F)}{\min\{d(E),d(F)\}}\,.$$

A pathwise connected metric measure space (X,μ) is said to be a *n-Loewner space,* if the Loewner function $\Phi_n(t)$ is positive for all $t>0$. Observe that, $\mathbb{R}^n$ and $\mathbb{B}^n \subset \mathbb{R}^n$ are Loewner spaces (see Theorem 8.2 and Example 8.24(a) in [2]). A domain D in $\mathbb{R}^n$ is called *QED-domain* if

$$M(\Gamma(E,F,\mathbb{R}^n)) \leqslant A \cdot M(\Gamma(E,F,D))$$

for some finite number $A \geqslant 1$ and all continua E and F in D. Given a domain D in X, a measurable function $Q : D \to [0,\infty]$, a compact set $K \subset D$ and numbers $A,\delta > 0$ denote by $\mathfrak{F}^{A,\delta}_{K,Q}(D)$ a family of all ring Q-homeomorphisms $f : D \to X'$ such that $D_f := f(D)$ is a compact QED-subdomain of X' with general A in the definition of a QED-domain and, in addition, $d'(f(K),\partial D_f) \geqslant \delta$.

Theorem. *Let (X,d,μ) and (X',d',μ') be metric spaces with metrics d and d' and locally finite Borel measures μ and μ', correspondingly. Let D be a domain in X in which the condition of the complete divergence of paths is satisfied. and let $f_n : D \to X'$, $n = 1,2,\ldots$, be a sequence of homeomorphisms of the class $\mathfrak{F}^{A,\delta}_{K,Q}(D)$ which converges to some mapping $f : D \to X'$ locally uniformly. Let X' be a n-Loewner space in which the relation $\mu(B_R) \leqslant C^*R^n$ holds for some constant $C^* \geqslant 1$, for some exponent $n>0$ and for all closed balls B_R of radius $R>0$. Let $K = \overline{G}$ and G is a compact subdomain of D. If $Q \in L^1(D)$, then f is a discrete in G.*

If in addition, X' is locally path connected, then f is open. Besides that, if all balls in X' are connected, and all closed balls in X are compact, then f is a homeomorphism.

R E F E R E N C E S

1. Martio, O., Ryazanov, V., Srebro, U., and Yakubov, E. *Moduli in modern mapping theory,* Springer Science + Business Media, LLC, New York (2009).

2. Heinonen, J. *Lectures on Analysis on metric spaces,* Springer Science+Business Media, New York (2001).