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SELECTION OF TARGET FUNCTION IN OPTICAL COATINGS SYNTHESIS PROBLEMS

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Abstract. The article presents general information on the use of optical coatings in various industries and analyzes the main approaches to optimizing optical filter structures. An approach to solving a class of optical coating synthesis problems is proposed, based on the formation of a new optimization model. The primary attention is paid to the formalization and analysis of the target function. To determine the quality of the optical coating, the deviation of the spectral characteristics from the required ones was estimated using the least squares, least absolute deviation, and minimum criteria. As a result, both smooth and two non-smooth target functions are proposed and analyzed. The peculiarities of their application in solving optimization problems related to optical coating synthesis are described, and corresponding numerical experiments are presented.

Keywords: optical coatings synthesis, wide bandpass filters, mathematical modeling, optimization, r-algorithm.

INTRODUCTION

Optical layered coatings have been used in a vast array of applications across different industries for many decades. They are used to modify the behaviour of light, enhancing the performance of optical devices in several ways. These coatings are commonly made of thin films of different materials that are deposited onto a substrate using various techniques, including sputtering, evaporation, and chemical vapour deposition [1]. One of the most prominent applications of optical layered coatings is in the field of optics. Optical lenses, filters, and mirrors are coated with thin layers of materials such as titanium dioxide, silicon dioxide, and aluminium to modify their refractive index, reflectivity, and transmission properties. These coatings help to reduce unwanted reflections, increase the light transmission, and improve colour accuracy, resulting in sharper, clearer images [2]. The film industry also relies heavily on optical coatings to improve the performance of cameras and lenses. Antireflective coatings applied to camera lenses reduce lens flare and ghosting, leading to crisper, higher-quality images. Similarly, polarizing filters are used to eliminate reflections and glare, resulting in better contrast and richer colours in the final footage. Optical layered coatings are also crucial in the medical field [3]. They are used to improve the performance of various medical devices, such as endoscopes, surgical lasers, and imaging systems. These coatings help to increase light transmission, reduce unwanted reflections, and improve the resolution and contrast of medical images, resulting in more accurate diagnoses and better treatment outcomes. In the field of electronics, optical layered coatings are used in the production of various displays, including LCDs and OLEDs [4]. These coatings help to increase the brightness and contrast of displays, reduce glare and reflections, and improve colour accuracy. They are also used in the production of solar panels to increase the efficiency of light absorption and conversion into electricity [5].

There are various approaches to optimizing the structures of optical layered coatings [6]. The trial-and-error method [7] involves manually adjusting the thickness and refractive index of each coating layer until the desired optical performance is achieved. However, this method can be time-consuming and does not always lead to an optimal coating design. Analytical methods use mathematical equations to calculate the thickness and refractive index of each coating layer. Some common analytical methods [8] are based on quarter-wave structures or structures that use bandwidth matching. These methods are relatively easy to use but do not always result in the optimal coating structure.

Numerical methods use computer algorithms to model the behavior of light waves within the coating and optimize the structure based on predefined criteria. Some common numerical methods include the transfer matrix method and the reverse wave analysis (RWA) method. The transfer matrix method [9] does not provide a natural way to model these optical properties, making it insufficient for synthesizing optical coatings. This method also assumes linear transformations, which do not account for light dispersion as it passes through materials. In optical coatings, materials are typically used where dispersion is a significant factor and must be considered in the design. The RWA method [10] can be very sensitive to initial conditions or input data. Even minor errors or inaccuracies in measurements or models can lead to incorrect results. However, these methods can be highly accurate and consider a wide range of structural criteria, but they are computationally complex [11].

Genetic algorithms [12] can be effective for the synthesis of optical coatings, but they may require a significant amount of computational resources and can be quite slow. The method of microstructured surfaces [13] uses structured microelements on the surface to create the desired optical properties. However, their production can be complex and require high-precision processing. Optical coatings created using the phase mask method [14] can be sensitive to changes in temperature, humidity, and mechanical stresses, leading to changes in their optical properties.

When using numerical methods, the choice of the objective function plays an important role. This work proposes several objective functions that can be used to optimize the parameters of optical coatings. One smooth and two non-smooth objective functions are presented. The effectiveness of their use is demonstrated with an example of a non-smooth objective function.

PROBLEM STATEMENT AND MATHEMATICAL MODEL

Multilayer optical coatings represent a structure consisting of N layers. The j-th layer is characterized by two parameters: the refractive index (n_j) and the geometric thickness (d_j) (Fig. 1). There are two main tasks associated with them. The first task, known as the direct or analysis task, involves determining the spectral characteristics (transmission, reflection, and absorption coefficients) of a known multilayer thin-film system based on the known characteristics of the coating. The task of calculating the characteristics of an interference coating is based on solving the stationary wave equation in the plane wave approximation. To date, a large number of computational schemes have been developed for calculating optical coatings. Perhaps the most common approach is based on calculating the tangential components of the electric and magnetic field vectors sequentially at all layer boundaries that form the coating. Introducing the matrix form of recording equations that connect the field amplitudes at adjacent boundaries allowed for a compact and consistent consideration of interference effects in layered structures of all types.

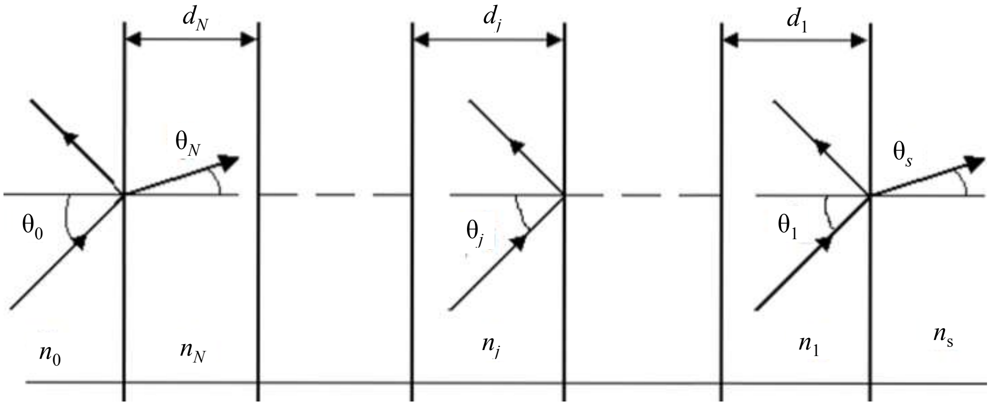

Fig. 1. Scheme of a light transmission through a multilayer optical structure

The second task, known as the inverse or synthesis task, involves determining the parameters of the multilayer optical structure that would optimally reproduce its predetermined spectral characteristics. In other words, the synthesis problem is to find such parameters of multilayered optical coating — refractive indices $\vec{n} = (n_1, n_2, \ldots, n_N)$, and geometric thicknesses of layers $\vec{d} = (d_1, d_2, \ldots, d_N)$ (N — number of layers), — under which, function, chosen to estimate transmittance factor quality, will be minimal in a given wavelength range $[\lambda_1, \lambda_2]$:

$$F^* = F(\vec{n}^*, \vec{d}^*) = \min_{\vec{n}, \vec{d}} F(\vec{n}, \vec{d}), \tag{1}$$

subject to

$$n_i^{\min} \le n_i \le n_i^{\max}, \; i = 1, 2, \ldots, N, \tag{2}$$

$$d_i^{\min} \le d_i \le d_i^{\max}, \; i = 1, 2, \ldots, N, \tag{3}$$

where F^* — minimum value of a coating target function.

Constrains (2), (3) have been imposed on the following parameters of multilayered optical coating — refractive indices and optical thicknesses. The refractive indices have been selected from the available coating-forming materials. Different sets of them can be created based on the spectral ranges of materials.

For visible and infrared rages, as a rule, the refractive index does not exceed 2.6. For the ultraviolet rage, materials with a higher refractive index can be used. Constraints (3) have been imposed on the geometric thickness of coating. The lower limit is tied to the application process, the upper limit, in the process of making multilayered optical coatings, as a rule, does not exceed the operating wavelength λ_0.

The value of the energy transmittance index for the electromagnetic wavelength λ through the multilayer optical structure should light fall on the surface at an angle θ_0 (Fig.1) has been calculated through the coefficients of the characteristic matrix $M(\vec{n},\vec{d},\lambda)$ as follows:

$$T(\vec{n},\vec{d},\lambda,\theta_0)=$$

$$=\frac{4}{2+\frac{p_0}{p_s}M_{11}^2(\vec{n},\vec{d},\lambda,\theta_0)+\frac{p_s}{p_0}M_{22}^2(\vec{n},\vec{d},\lambda,\theta_0)+p_0p_sM_{12}^2(\vec{n},\vec{d},\lambda,\theta_0)+\frac{1}{p_0p_s}M_{21}^2(\vec{n},\vec{d},\lambda,\theta_0)},$$

where $p_0=n_0\cos\theta_0$ and $p_s=n_s\cos\theta_s$ — for TE wave (s-polarization); $p_0=\dfrac{n_0}{\cos\theta_0}$ and $p_s=\dfrac{n_s}{\cos\theta_s}$ — for TE wave (p -polarization); θ_0 — angle of incidence; θ_s — angle of reflection; n_0, n_s — refractive indices of an environment and a substrate, accordingly.

The characteristic matrix of the *N*-layer structure is equal to the product of the matrices of each of the layers [15]:

$$M(\vec{n},\vec{d},\lambda,\theta_0)=$$

$$=M(n_N,d_N,\lambda,\theta_N)M(n_{N-1},d_{N-1},\lambda,\theta_{N-1})\cdots M(n_2,d_2,\lambda,\theta_2)M(n_1,d_1,\lambda,\theta_1),$$

where the characteristic matrix of the layer equals

$$M(n,d,\lambda,\theta)=\begin{matrix}\cos\delta(n,d,\lambda,\theta) & -\frac{i}{n}\sin\delta(n,d,\lambda,\theta), \\ -in\sin\delta(n,d,\lambda,\theta) & \cos\delta(n,d,\lambda,\theta),\end{matrix}$$

$\delta(n,d,\lambda,\theta)=\dfrac{2\pi nd\cos\theta}{\lambda}$ — phase thickness of the layer; θ —angle of incidence.

Angles of incidence for each layer follow the Snell's law and can be easily calculated according to the ratio:

$$n_0\sin\theta_0= n_1\sin\theta_1=n_2\sin\theta_2=\ldots=n_j\sin\theta_j=\ldots=n_N\sin\theta_N=n_s\sin\theta_s.$$

If $\theta_0=0$, then the value of transmittance factor for the *N*-layer optical structure can be calculated using the following formula

$$T(\vec{n},\vec{d},\lambda)=$$

$$=\frac{4}{2+\frac{n_0}{n_s}M_{11}^2(\vec{n},\vec{d},\lambda)+\frac{n_s}{n_0}M_{22}^2(\vec{n},\vec{d},\lambda)+n_0n_sM_{12}^2(\vec{n},\vec{d},\lambda)+\frac{1}{n_0n_s}M_{21}^2(\vec{n},\vec{d},\lambda)},$$

where the characteristic matrix of the *N*-layer structure is written as

$$M(\vec{n},\vec{d},\lambda)=M(n_N,d_N,\lambda)M(n_{N-1},d_{N-1},\lambda)\cdots M(n_2,d_2,\lambda)M(n_1,d_1,\lambda),$$

and characteristic matrix of one layer is given by

$$M(n,d,\lambda)=\begin{matrix}\cos\frac{2\pi nd}{\lambda} & -\frac{i}{n}\sin\frac{2\pi nd}{\lambda} \\ -in\sin\frac{2\pi nd}{\lambda} & \cos\frac{2\pi nd}{\lambda}\end{matrix}.$$

It should be noted, that characteristic matrix of the multilayered optical structure meets following condition

$$det(M(\vec{n},\vec{d},\lambda)) = 1. \tag{4}$$

This follows from the fact that the characteristic matrix of each layer has the same property

$$det(M(n_i, d_i, \lambda)) = 1, \ i = 1, 2, \ldots, N.$$

Property (4) has a simple physical meaning. If an electromagnetic wave propagates in N media that do not absorb its energy, then an arbitrarily combined (of these N media) medium will not absorb the energy of the electromagnetic wave.

OPTICAL COATING TARGET FUNCTIONS AND THEIR USE

The following coating target functions can be chosen to solve the synthesis problem (1)–(3):

$$F_1(\vec{n},\vec{d}) = \frac{1}{L}\sum_{i=1}^{L} w_i (T(\vec{n},\vec{d},\lambda_i) - T_{ideal}(\lambda_i))^2, \tag{5}$$

$$F_2(\vec{n},\vec{d}) = \frac{1}{L}\sum_{i=1}^{L} w_i \left|T(\vec{n},\vec{d},\lambda_i) - T_{ideal}(\lambda_i)\right|, \tag{6}$$

$$F_3(\vec{n},\vec{d}) = \max_{\mathrm{i=1,\ldots,L}} w_i \left|T(\vec{n},\vec{d},\lambda_i) - T_{ideal}(\lambda_i)\right|, \tag{7}$$

where w_i — weighting coefficients, which determine the input on the objective function at wavelength λ_i; L — the number of grid points on the spectral interval between λ_1 and λ_2; $T(\vec{n},\vec{d},\lambda_i)$ — the value of the transmission index for parameters $(\vec{n},\vec{d})$ and at wavelength λ_i; $T_{ideal}(\lambda_i)$ — the value of the transmission index at wavelength λ_i.

Coating target functions (5)–(7) have been described below. Function $F_1(\vec{n},\vec{d})$ sets the weighted standard deviation of the transmittance indices from the required for the selected L values of wavelengths. This function is smooth, so gradient methods, quasi-Newton methods and zero-order methods (use only the values of the objective function) can be used to minimize it. Function $F_2(\vec{n},\vec{d})$ sets the weighted sum of deviations from the mean with respect to the selected L. Function $F_3(\vec{n},\vec{d})$ specifies deviation under minimax control (Chebyshev criterion). The functions $F_2(\vec{n},\vec{d})$ and $F_3(\vec{n},\vec{d})$ are non-smooth, so Shore r-algorithms and zero-order methods can be used to minimize them.

When solving the antireflective coating substrate problem, the values of $T_{ideal}(\vec{n},\vec{d},\lambda_i)$ are constand and equal to unity. With regard to afford mentioned, the objective functions takes the form:

$$F_1(\vec{n},\vec{d}) = \frac{1}{L}\sum_{i=1}^{L} w_i (T(\vec{n},\vec{d},\lambda_i) - 1)^2,$$

$$F_2(\vec{n},\vec{d}) = \frac{1}{L}\sum_{i=1}^{L} w_i \left|T(\vec{n},\vec{d},\lambda_i) - 1\right|,$$

$$F_3(\vec{n},\vec{d}) = \max_{\mathrm{i}=1,\ldots,\mathrm{L}} w_i \left| T(\vec{n},\vec{d},\lambda_i) - 1 \right|.$$

Given that the value of transmittance factor is less than unity, the function $F_2(\vec{n},\vec{d})$ can be expressed in the following form

$$F_2(\vec{n},\vec{d}) = \frac{1}{L}\sum_{i=1}^{L} w_i \left| T(\vec{n},\vec{d},\lambda_i) - 1 \right| = \frac{1}{L}\sum_{i=1}^{L} w_i - \frac{1}{L}\sum_{i=1}^{L} w_i T(\vec{n},\vec{d},\lambda_i),$$

and will be smooth, when solving the antireflective coating substrate problem. In the similar fashion, the function $F_3(\vec{n},\vec{d})$ can be expressed in the following form

$$F_3(\vec{n},\vec{d}) = \max_{i=1,\ldots,L} w_i \left| T(\vec{n},\vec{d},\lambda_i) - 1 \right| = \max_{i=1,\ldots,L} w_i (1 - T(\vec{n},\vec{d},\lambda_i)),$$

But in contrast to the function $F_2(\vec{n},\vec{d})$, it`s non-smooth.

If all $w_i = 1$, we obtain following objective functions:

$$F_1(\vec{n},\vec{d}) = \frac{1}{L}\sum_{i=1}^{L} (T(\vec{n},\vec{d},\lambda_i) - 1)^2,$$

$$F_2(\vec{n},\vec{d}) = \frac{1}{L}\sum_{i=1}^{L} \left| T(\vec{n},\vec{d},\lambda_i) - 1 \right| = 1 - \frac{1}{L}\sum_{i=1}^{L} T(\vec{n},\vec{d},\lambda_i),$$

$$F_3(\vec{n},\vec{d}) = \max_{\mathrm{i}=1,\ldots,\mathrm{L}} \left| T(\vec{n},\vec{d},\lambda_i) - 1 \right| = \max_{\mathrm{i}=1,\ldots,\mathrm{L}} (1 - T(\vec{n},\vec{d},\lambda_i)).$$

In a number of studies problems of wide bandpass optical coatings synthesis have been reviewed as maximization problems for similar deviations, and not for the maximum transmittance, but for the minimum possible, i.e. zero value of the transmittance [16]. For weighted standard deviation, there is an alternative, where the maximization problem can be described as

$$\max_{\vec{n},\vec{d}} \left(\mathrm{F}(\vec{n},\vec{d}) = \frac{1}{L}\sum_{i=1}^{L} T^2(\overrightarrow{n(\lambda_i)},\vec{d},\lambda_i) \right), \tag{8}$$

subject to (2) and (3).

In a similar way, for weighted sum of deviations from the mean this problem can be described as

$$\max_{\vec{n},\vec{d}} \left(\mathrm{F}(\vec{n},\vec{d}) = \frac{1}{L}\sum_{i=1}^{L} T(\overrightarrow{n(\lambda_i)},\vec{d},\lambda_i) \right), \tag{9}$$

subject to (2) and (3).

And for deviation under minimax control (Chebyshev criterion) is as follows

$$\max_{\vec{n},\vec{d}} \left(\mathrm{F}(\vec{n},\vec{d}) = \min_{\mathrm{i}=1,\ldots,\mathrm{L}} T(\overrightarrow{n(\lambda_i)},\vec{d},\lambda_i) \right). \tag{10}$$

subject to (2) and (3).

For these models, which use target functions (8)–(10), it is assumed that there may be a refractive index dispersion. Accordingly, the value of the refractive index is a function of wavelength and function is defined using approximation Zellmeier formula

$$n_i(\lambda) = \sqrt{A_i + \frac{B_i}{\lambda^2} + \frac{C_i}{\lambda^4} + D_i\lambda^2 + E_i\lambda^4} \ ,$$

where A_i, B_i, C_i, D_i, E_i — parameters for refraction index model in the presence of dispercion. Optical materials can be described either by the values of the dispersion formula coefficients, or directly by the values of the refractive index for different wavelengths. For many optical materials, this information is available in databases. Also, during the study, one layer can be considered smooth or partially inhomogeneous [17].

Problem (1)–(3) is multiextremal. It contains 2N variables, where the first N variables are the refractive indices of the layers, the second N variables are the geometric thicknesses of the layers. Bilateral constraints on variables are set by conditions (2)–(3). The local minima of the problem (1)–(3) often provide the required approximation accuracy and have implementable coating parameters. Such solutions are often called quasi-optimal. In this work we decided to follow up on the suggested term, so by quasi-optimal solutions we will always mean such local extremums of problem (1)–(3), for which the found coating parameters are practically feasible.

Problem (1)–(3) can be modeled as unconstrained optimization by using transition from one variables to another

$$x_j = x_j^{\min} + (x_j^{\max} - x_j^{\min}) \sin^2 z_j, \tag{11}$$

$$x_j = \frac{x_j^{\max} z_j^2 + x_j^{\min}}{z_j^2 + 1}, \quad j = 1, \ldots, N. \tag{12}$$

Thus, a solution for each parameter can be found at infinity. An objective function has been complicated by this. Formula variables (11) provide a smoother change of the formed surface and have less abrupted transition in comparison to another formula (12). On the other hand, the transition to unconstrained optimization by formula (11) requires the calculation of the value of $\arcsin(x)$, which is a rather time-consuming operation. For the approach used in this paper, this applies to both the values of geometric thicknesses and refractive indices. To do this, the minimum and maximum refractive indices must be selected.

As the number of layers increases, more parameters for reduction of the target functions $F(\vec{n}, \vec{d})$ value in the optimization problems of optical coatings synthesis, can be obtained. Therefore, it is necessary to clarify the criterion for termination of the search process for solving optimization problem (1)–(3). This goal can be archived by looking for ε solution:

$$\left| F(\vec{n}_\varepsilon^*, \vec{d}_\varepsilon^*) - F^* \right| \le \varepsilon. \tag{13}$$

In case of minimization problem — it will be inequation $F(\vec{n}_\varepsilon^*, \vec{d}_\varepsilon^*) - F^* \le \varepsilon$, and in the case of maximization problem — $F^* - F(\vec{n}_\varepsilon^*, \vec{d}_\varepsilon^*) \le \varepsilon$.

The introduction of inequality (13) into the optimization model has been caused by two factors. First, there are a large number of quasi-optimal solutions that can have a design implementation. Secondly, it is often impossible to achieve an exact approximation of predetermined spectral characteristics. The spectral characteristics of the optical coating are analytical functions and can be differentiated an infinite number of times [11]. Accordingly, if the idealized characteristic is constant or has gaps, then exact approximation cannot be obtained. From a practical point of view, the definition of the problem should also include a condition of limiting the number of layers, which would serve as a criterion for termination of the search process, and can serve for correction of a sufficiently small value of ε.

An additional condition associated with the manufacture of optical coating selects one design from a variety of solutions that meet criterion (13), and accordingly, the second Hadamard condition will be met. This condition must also take into account the characteristics of the selected materials, their interaction with each other.

The application of the Monte Carlo method allows choosing the most fault tolerant design solutions [18]. Therefore, for the chosen optical coating, the condition must be met that a slight change in the input parameters will also satisfy criterion (8), and, respectively, will satisfy the third Hadamard condition.

COMPUTATIONAL EXPERIMENT

The developed approach has been applied to improve the behaviour of existing wide bandpass coatings. For this purpose, we used Shor's R-algorithm [11; 19] with coating target function represented as

$$F(\vec{n},\vec{d}) = \min_{\lambda_1 \le \lambda \le \lambda_2} \sum_{i=1}^{L} \left|1 - T(\vec{n},\vec{d},\lambda_{(i)}),\right|$$

where $[\lambda_1,\lambda_2]$ — wavelength range under study; L — number of points in the wavelength range from λ_1 to λ_2. In this section, the chosen value of L equals $\lambda_2 - \lambda_1 + 1$, i.e. in the objective function, each integer-value of the interval was considered $[\lambda_1,\lambda_2]$.

Let us demonstrate application of the proposed optimization approach on a practical example. For this, we will use three optical coatings known in the industry.

In wavelength range between 450 and 800, value of the first coating target function $F(\vec{n},\vec{d}) = 1.404$ (curve 1 — parameters of the optical coating known in the industry $3.76{\cdot}n_1d_1{=}3.76{\cdot}n_2d_2{=}0.455{\cdot}n_3d_3{=}n_4d_4{=}0.25{\cdot}\lambda_0$, $n_1{=}2.0$, $n_2{=}1.37$, $n_3{=}2.0$, $n_4{=}1.37$), and for the second — $F(\vec{n},\vec{d})$ = 0.838 (curve 2 — parameters, which have been calculated in this article $6.58{\cdot}n_1d_1{=}4.06{\cdot}n_2d_2{=}0.441{\cdot}n_3d_3{=}0.944{\cdot}n_4d_4{=}0.25{\cdot}\lambda_0$, $n_1{=}2.1$, $n_2{=}1.35$, $n_3{=}1.9$, $n_4{=}1.35$). Accordingly, value of the coating target function $\mathrm{F}(\vec{n},\vec{d})$ has been improved by 40% (Fig. 2).

Graph of the coating target function can be easily assessed, if we will fix all parameters, except two (except geometric thicknesses of third and fourth layers, have been fixed, for optical coating with parameters $0.153{\cdot}n_1d_1{=}0.25{\cdot}n_2d_2{=}$ $=0.25{\cdot}\lambda_0$, $n_1{=}1.35$, $n_2{=}1.9$, $n_3{=}1.35$, $n_4{=}2.1$ in the case of antireflection coating application with refractive index $n_s = 1.52$). As can be seen in the Fig. 3, even the part of the graph let us assume that this graph has a ravine-type shape. Let's consider the sevenlayer antireflection coating, consisting of alternating layers

(1.35 and 2.1), for which layer optical depths in respect to λ_0 are as follows — 0.05 : 0.071 : 0.062 : 0.257 : 0.018 : 0.12 : 0.2, for which all derived optimal parameters, except geometric thicknesses of sixth and seventh layers, has been fixed. Resulting graph (Fig. 6) clearly shows that graph of the estimated target function has, indeed, a ravine-type shape. It has fixed all the optimal parameters, except geometric thicknesses of sixth and seventh layers, have been fixed, for sevenlayer antireflection coating, consisting of alternating layers with refractive indices 1.35 and 2.1, layer optical depths of the first five layers with respect to λ_0 are as follows — 0.05 : 0.071 : 0.062 : 0.257 : 0.018.

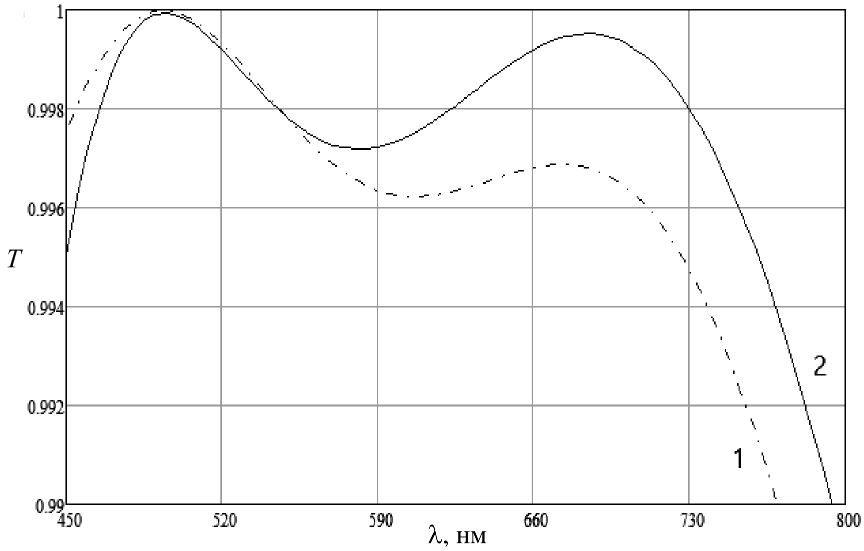

Fig. 2. Wide bandpass filter transmittance curve in the case of antireflection coating application with refractive index n_s=1.51

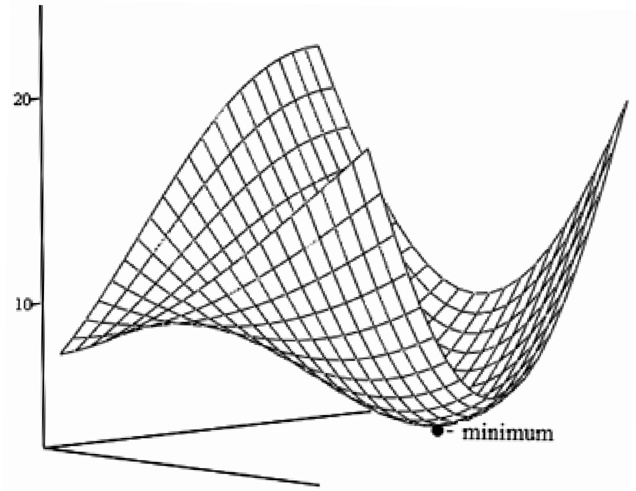

Fig. 3. Graph of the quality function of the four-layer coating

In wavelength range between 450 and 750, value of the first coating target function $F(\vec{n},\vec{d}) = 0.665$ (curve 1 — parameters of the optical coating known in the industry, layer optical depths with respect to λ_0 are as follows — 0.064 : 0.038 : 0.401 : 0.032 : 0.084 : 0.459 : 0.229), and for the second —

$F(\vec{n},\vec{d}) = 0.324$ (curve 2 — parameters, which have been calculated in this article, layer optical depths with respect to λ_0 are as follows — 0.087 : 0.03 : 0.315 : 0.043 : 0.113 : 0.48 : 0.22). Accordingly, value of the coating target function $F(\vec{n},\vec{d})$ has been improved by more than 50% (Fig. 4).

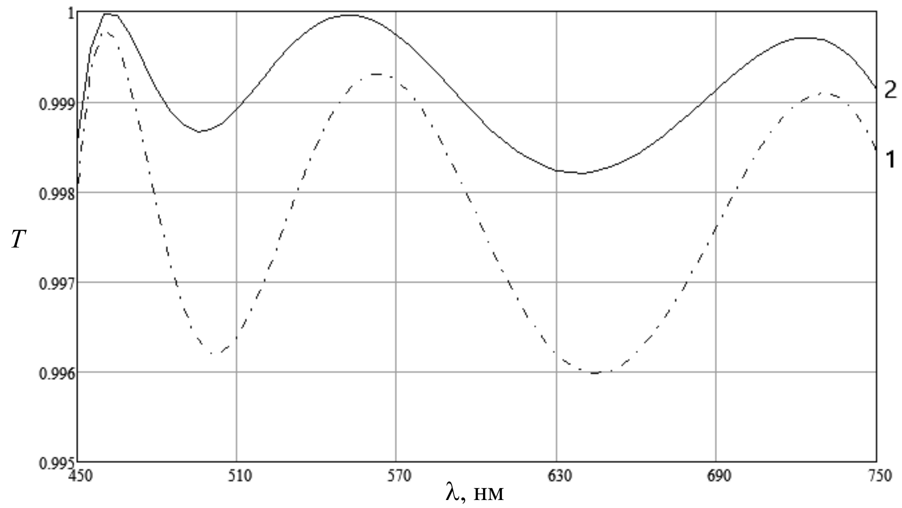

Fig. 4. Transmittance curve for sevenlayer antireflection coating, consisting of alternating layers (1.35 and 2.1) of substrate with refractive index n_s=1.52

It should be noted, that for gradient methods, the use of this objective function gives a less effective result. For these methods, one must use the target function (10).

In wavelength range between 450 and 750, value of the first coating target function $F(\vec{n},\vec{d}) = 0.953$ (curve 1 — parameters of the optical coating known in the industry, layer optical depths with respect to λ_0 are as follows — 0.06 : 0.02 : 0.35 : 0.02 : 0.07 : 0.42 : 0.21), and for the second — $F(\vec{n},\vec{d}) = 0.478$ (2 — parameters, which have been calculated in this article, layer optical depths with respect to λ_0 are as follows — 0.05 : 0.071 : 0.062 : 0.257 : 0.018 : 0.12 : 0.2). Accordingly, value of the coating target function $F(\vec{n},\vec{d})$ has been improved by almost 50% (Fig. 5).

CONCLUSIONS

This paper describes three types of target functions, which can be used for solving optimization problems of optical coatings synthesis. Their reduction to the problems of unconstrained minimization of smooth and non-smooth functions has been described and the peculiarities of the transition to new variables for each of the proposed models has been investigated. The following computer implementations can be used to accelerate solving optical coating synthesis problems: tabulation of values of trigonometric functions, fast matrix multiplication and the use of an efficient method for one-dimensional optimization.

A computational experiment has been performed, in which the target function in the form of the weighted sum of deviations from the mean was taken and spectral characteristics of the three available wide bandpass antireflection filters has been improved by using the r-algorithm for optimization. For one of the wide bandpass antireflective coatings, the target function was improved by 40%, and for the other two, the target function was improved by 50%.

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ВИБІР ФУНКЦІЇ ЯКОСТІ В ЗАДАЧАХ СИНТЕЗУ ОПТИЧНИХ ПОКРИТТІВ
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Анотація. Наведено загальні відомості про використання оптичних покриттів у різних галузях промисловості та проаналізовано основні підходи до оптимізації структур оптичних фільтрів. Запропоновано підхід до вирішення класу задач синтезу оптичних покриттів, заснований на формуванні нової оптимізаційної моделі. Основну увагу приділено формалізації та аналізу цільової функції. Для визначення якості оптичного покриття використано оцінку відхилення спектральних характеристик від необхідних за критеріями найменших квадратів, найменших абсолютних відхилень і мінімаксу. У результаті запропоновано та досліджено як гладку, так і дві негладкі цільові функції. Описано особливості їх застосування в розв'язуванні оптимізаційних задач синтезу оптичних покриттів та наведено відповідні числові експерименти.

Ключові слова: синтез оптичних покриттів, широкосмугові фільтри, математичне моделювання, оптимізація, *r*-алгоритм.