

On convergence theorems for ring Q -homeomorphisms with integrable majorant

Nataliya Ilkevych

Zhytomyr Ivan Franko State University

Denys Romash

Zhytomyr Ivan Franko State University
e-mail: d.romash@num8erz.eu

Evgeny Sevost'yanov

Zhytomyr Ivan Franko State University; Institute of Applied Mathematics and Mechanics of NAS of Ukraine
e-mail: esevostyanov2009@gmail.com
ORCID: 0000-0001-7892-6186

Abstract. Everywhere further, $D \subset \mathbb{R}^n$, $n > 2$, is a domain in $\mathbb{R}^n$ with the Euclidean metric $|x - y|$ and the Lebesgue measure m . For $x_0 \in D$ and $r > 0$ put $S(x_0, r) = \{x \in \mathbb{R}^n : |x - x_0| = r\}$, and for a compact $K \subset D$ define $d(K, \partial D) = \inf\{|x - y| : x \in K, y \in \partial D\}$. Let $Q : D \rightarrow [0, \infty]$ be measurable.

A homeomorphism $f : D \rightarrow \mathbb{R}^n$ is called a ring Q -homeomorphism at $x_0 \in D$ with respect to (p, q) -moduli if for every $0 < r_1 < r_2 < d(D)$,

$$M_p(f(\Gamma(S(x_0, r_1), S(x_0, r_2), A(x_0, r_1, r_2)))) \leq \int_{A(x_0, r_1, r_2) \cap D} Q(x) \eta^q(|x - x_0|) dm(x) \quad (3)$$

for all measurable $\eta : (r_1, r_2) \rightarrow [0, \infty]$ satisfying

$$\int_{r_1}^{r_2} \eta(r) dr \geq 1, \quad (4)$$

where $A(x_0, r_1, r_2) = \{x \in D : r_1 < |x - x_0| < r_2\}$.

A domain D is a QED -domain if there exists $A \geq 1$ such that

$$M(\Gamma(E, F, \mathbb{R}^n)) \leq A \cdot M(\Gamma(E, F, D)) \quad (5)$$

for all continua $E, F \subset D$.

Given $A \geq 1$, $\delta > 0$ and $K \subset D$ compact, denote by $\mathfrak{F}_{K,Q}^{A,\delta}(D)$ the family of all mappings $f : D \rightarrow \mathbb{R}^n$ satisfying (1)–(2), such that $D_f := f(D)$ is a QED -domain with constant A in (3) and $d(f(K), \partial D_f) > \delta$.

The main result of the corresponding paper establishes the lower estimate

$$|f(x) - f(y)| \geq C_1 \exp \left\{ - \frac{\|Q\|_1 A}{C |x - y|^n} \right\}, \quad x, y \in K, \quad (6)$$

valid for all $f \in \mathfrak{F}_{K,Q}^{A,\delta}(D)$. Using this estimate, we obtain the following convergence theorem.

Theorem. Let $D \subset \mathbb{R}^n$, $n > 2$, and let $f_m : D \rightarrow \mathbb{R}^n$ be homeomorphisms of the class $\mathfrak{F}_{K,Q}^{A,\delta}(D)$ converging locally uniformly in D to f . If $K = \overline{G}$ and G is a compact subdomain of D , then for $Q \in L^1(D)$ the limit mapping f is a homeomorphism in G .

Keywords: ring Q -homeomorphisms; lower distance estimates; QED -domains; convergence theorems.