On Carathéodory theorem for Orlicz-Sobolev classes

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All definitions and notions used below may be found in [1]. The following notation is used: the set of prime ends corresponding to the domain D is denoted by E_D, and the completion of the domain D by its prime ends is denoted by $\overline{D}_P$. Below $C(f, \partial D)$ denotes the cluster set of f on ∂D. The following result holds.

Theorem 1. *Let $n-1<\alpha\leqslant n$, let D and D' be bounded domains in $\mathbb{R}^n$, $n\geqslant 3$, let $Q:D\rightarrow[0,\infty]$ be a Lebesgue measurable function and let $\varphi:[0,\infty)\rightarrow[0,\infty)$ be an increasing function. Let D be a regular domain and let $f:D\rightarrow D'$ be an open discrete mapping in $W^{1,\varphi}_{\rm loc}(D)$, $f(D)=D'$. In addition, assume that $C(f,\partial D)\subset E_*$ for some closed (in the topology of $\mathbb{R}^n$) set $E_*\subset\overline{D'}$ and $f^{-1}(E_*)=E$ for some closed (in the topology of D) subset $E\subset D$. In addition, assume that: 1) the set E is nowhere dense in D and D is finitely connected on $E\cup\partial D$, i.e., for any $z_0\in E\cup\partial D$ and any neighborhood $\widetilde{U}$ of z_0 there is a neighborhood $\widetilde{V}\subset\widetilde{U}$ of z_0 such that $(D\cap\widetilde{V})\setminus E$ consists of finite number of components, 2) for any $P\in E_D:=\overline{D}_P\setminus D$ and for every neighborhood U of P in $\overline{D}_P$ there is a neighborhood $V\subset U$ in $\overline{D}_P$ of P such that $V\cap D$ is connected and $(V\cap D)\setminus E$ consists at most of m components, $1\leqslant m<\infty$, 3) all components of the set $D'\setminus E_*$ have a strongly accessible boundary with respect to α-modulus, 4) the function φ satisfies the following Calderon condition*

$$\int\limits_{1}^{\infty}\left(\frac{t}{\varphi(t)}\right)^{\frac{1}{n-2}}\,dt<\infty\,. \tag{1}$$

5) Assume that $K_{I,\alpha}(x,f)\leqslant Q(x)$ a.e. whenever $K_{I,\alpha}(x,f)$ is the inner dilatation of f of the order α and, in addition, the condition $\int\limits_{0}^{\delta(b)}\frac{dt}{t^{\frac{n-1}{\alpha-1}}q_b^{\prime\,\frac{1}{\alpha-1}}(t)}=\infty$ holds for every point $b\in\partial D$ and some $\delta(b)>0$, where $q^{\prime}_b(t)$ denotes the integral average of the function $Q^{\prime}$ under the sphere $S(b,t)$, while $Q^{\prime}(x)=\begin{cases}Q(x), & Q(x)\geqslant 1\,,\\ 1, & Q(x)<1\,.\end{cases}$ Then f has a continuous extension $\overline{f}:\overline{D}_P\rightarrow\overline{D'}$, moreover, $\overline{f}(\overline{D}_P)=\overline{D'}$.

This result is published in [2].

References

[1] Martio O., Ryazanov V., Srebro U. and Yakubov E. *Moduli in Modern Mapping Theory.* Springer Science + Business Media, LLC : New York, 2009.
[2] Kovba Z., Sevost'yanov E. *On Carathéodory prime ends extension for unclosed Orlicz-Sobolev classes.* https://arxiv.org/abs/2604.15026.

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