On Koebe-Bloch theorem under some integral constraints

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All necessary definitions may be found in [1]. Let $A = A(x_0, r_1, r_2) = \{x \in \mathbb{R}^n : r_1 < |x-x_0| < r_2\}$, let $B(x_0, r_0) = \{x \in \mathbb{R}^n : |x - x_0| < r_0\}$, let $M_p(\cdot)$ be the modulus of family of paths of the order $p \geqslant 1$, and let $dm(y)$ be an element of the Lebesgue measure in $\mathbb{R}^n$, $n \geqslant 2$. We say that f *satisfies the inverse Poletsky inequality at a point* $y_0 \in \overline{f(D)} \setminus \{\infty\}$ *with respect to p-modulus,* if the relation

$$M_p(\Gamma_f(y_0, r_1, r_2)) \leqslant \int\limits_{A(y_0,r_1,r_2)\cap f(D)} Q(y) \cdot \eta^p(|y - y_0|)\, dm(y) \qquad (1)$$

holds for any $0 < r_1 < r_2 < r_0 < \infty$ and any Lebesgue measurable function $\eta : (r_1, r_2) \to [0, \infty]$ such that $\int\limits_{r_1}^{r_2} \eta(r)\, dr \geqslant 1$. Given $p \geqslant 1$, a non-decreasing function $\Phi\colon \overline{\mathbb{R}^+} \to \overline{\mathbb{R}^+}$, $a, b \in D$, $a \neq b$, $\delta > 0$ we denote by $\mathfrak{F}^{\Phi,p}_{a,b,\delta}(D)$ the family of all open discrete mappings $f : D \to \mathbb{R}^n$, $n \geqslant 2$, for which there exists a Lebesgue measurable function $Q = Q_f : \mathbb{R}^n \to [0, \infty]$ satisfying relations (1) at any point $y_0 \in \overline{\mathbb{R}^n}$ with $\int\limits_{\mathbb{R}^n} \Phi(Q_f(y)) \cdot \frac{dm(y)}{(1+|y|^2)^n} < \infty$ such that $h(f(a), f(b)) \geqslant \delta$, where h is a chordal metric in $\overline{\mathbb{R}^n} := \mathbb{R}^n \cup \{\infty\}$. The following statement holds.

Theorem 1. *Let $n \geqslant 2$, $p \in (n - 1, n]$ and let D be a domain in $\mathbb{R}^n$. Let also $\Phi : \overline{\mathbb{R}^+} \to \overline{\mathbb{R}^+}$ be an increasing convex function that satisfies the condition $\int\limits_{\delta}^{\infty} \frac{d\tau}{\tau(\Phi^{-1}(\tau))^{\frac{1}{p-1}}} = \infty$ for some $\delta > \Phi(0)$. Assume that, the family $\mathfrak{F}^{Q,p}_{a,b,\delta}(D)$ is equicontinuous at a and b. Then for every compactum K in D and for every $0 < \varepsilon < \operatorname{dist}(K, \partial D)$ there exists $r_0 = r_0(\varepsilon, K) > 0$ which does not depend on f, such that $f(B(x_0, \varepsilon)) \supset B_h(f(x_0), r_0)$ for all $f \in \mathfrak{F}^{\Phi,p}_{a,b,\delta}(D)$ and all $x_0 \in K$, where $B_h(f(x_0), r_0) = \{w \in \overline{\mathbb{R}^n} : h(w, f(x_0)) < r_0\}$.*

The mentioned result is established in [2].

References

[1] Martio O., Ryazanov V., Srebro U. and Yakubov E. *Moduli in Modern Mapping Theory.* Springer Science + Business Media, LLC : New York, 2009.

[2] Ilkevych N., Sevost'yanov E., Targonskii V. On Koebe's theorem for mappings with integral constraints. https://arxiv.org/abs/2509.02008.

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