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STEM-BASED DESIGN, AI TOOLS, AND PROBLEM TYPIFICATION IN SCHOOL STUDENTS' CONSTRUCTIVE LEARNING OF PARALLEL TRANSLATION

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Abstract. The transformation of geometric figures is one of the important topics in the school geometry course. Their study was primarily focused on the classical analytical representation. At the same time, the constructive component has been considered to a lesser extent. However, it allows for a more visual representation of the geometric meaning of each transformation. Thus, the task was set to use the example of the operation of parallel translation of figures, relying on the constructive nature of geometry to reveal important components in the formation of students' understanding of such transformations. The main body of the article presents examples supported by diagrams and worked solutions. On this basis, the authors illustrate how, in their view, the study of geometric transformations may be modified by shifting the emphasis toward geometric constructions, the practical implementation of student STEM projects, and the use of artificial intelligence technologies. The concluding section summarizes the material and outlines directions for further research on this topic, including the prospects for implementing the proposed theoretical model in educational practice.

Keywords: constructive geometry; parallel translation; AI technologies in education; STEM; typification of planimetric problems.

1. Introduction

The teaching of geometry, as well as Mathematics in general, remains a relevant topic of research for teachers and methodologists. One of the key factors influencing current discussions is the rapid development of digital technologies, particularly artificial intelligence tools. Educational systems must respond to these changes and adapt instructional practices accordingly. In this context, the study of geometric figures and their transformations is of practical importance. Let us focus in more detail on geometric transformations.

In many school curricula, geometric transformations receive limited attention, although they play a central role in understanding geometry as a discipline. The authors suggest that this limitation may be addressed by strengthening the constructive component of geometry instruction, encouraging STEM-based student projects, and integrating modern digital technologies.

The main aim of this article is to examine the study of parallel translation as an example of a geometric transformation and to highlight key components of its constructive learning. The discussion is based on the constructive and practice-oriented nature of geometry.

The methodology of the study includes general scientific methods, such as analysis and systematization of approaches to teaching geometric transformations in general secondary education. The results are presented in the form of solved construction problems accompanied by mathematical justification. According to the authors' view, this format helps illustrate how problem typification, the STEM approach, and AI tools may support students' understanding of geometric transformations.

2. Literature review

The scientific literature presents various approaches to introducing schoolchildren to this topic in order to demonstrate its significance. In particular, Gulkilik H. studied the role of virtual manipulations in high school students' understanding of geometric transformations (Gulkilik, 2016). In turn, Malatjie F. and Machaba F. investigated the impact of using concept maps in the educational process on students' understanding of this topic (Malatjie & Machaba, 2019). A slightly different emphasis is placed in the work of Hollebrands, K. F. The author considered the impact of using the Geometer's Sketchpad technological tool in teaching geometry on students' understanding of transformations on a plane or in space (Hollebrands, 2003). The formation of students' skills in constructing mathematical generalizations when studying facts related to movement on a plane or in space, symmetry about an axis, or similarity of figures is revealed by Yao X. and Manouchehri A. (Yao & Manouchehri, 2019).

The use of STEM approaches is also appropriate when teaching students both the topic in question and geometry in general. In particular, the use of STEM and gamification in studying the subject “Geometry” is discussed in the work of scientists Moral-Sánchez S. N., Sánchez-Compaña M. T., and Romero I. (Moral-Sánchez et al., 2022). In contrast, a team of authors consisting of Adnan N. A. S., Osman S., Kumar J. A., Jambari H., and Talib C. A. (2022) studied the influence on the formation of students' interest in STEM projects by examining geometry using augmented reality technologies. In their work, Nindiasari H., Pranata M. F., Sukirwan S., Sugiman S., Fathurrohman M., Ruhimat A., and Yuhana Y. propose using these same technologies to improve students' skills in solving geometric problems (Nindiasari et al., 2024).

Recent research increasingly addresses the use of artificial intelligence technologies in the teaching of geometry and mathematics. This is because the relevant tools have become widely available to the general public and are now commonly used, including in the educational process. Generative language models may support individualized learning, but they also raise concerns related to potential misuse and its long-term consequences. Although no universal solution exists, educators continue to explore responsible approaches to the use of these technologies. In particular, Şener, A., Poçan, S., and Ergen, B. (2025) see the use of deep learning models as a way to improve geometry education.

Numerous studies examine the broader impact of AI technologies on mathematics education. Shin (2020) discusses the potential of artificial intelligence tools for supporting individualized instruction and enhancing the objectivity of assessment in mathematics education. Uygun et al. (2024) argue that combining artificial intelligence with virtual and augmented reality tools may foster spatial reasoning and geometric visualization skills among students.

A review of the literature indicates that no consensus currently exists regarding the most effective technologies or instructional approaches for teaching geometric transformations. The constructive component remains a fundamental aspect of geometry; therefore, geometric transformations may also be examined from the perspective of constructive methods.

3. Typification of geometric construction problems in the study of parallel translation

A comprehensive study of geometry requires a clear understanding of transformations both in the plane and in space. Of course, the classic approach to studying this issue in school mathematics courses is entirely appropriate, but it is not without its drawbacks. One such limitation is the excessive emphasis on analytical representations of transformations. As a result, the visual and constructive aspects of geometry receive less attention, which may hinder students' understanding of the topic.

A potential way to address this issue is to focus on studying this topic through planimetric construction problems.

We clearly indicate the non-trivial nature of this type of problem. The solution of such problems typically includes four stages: analysis, construction, proof, and investigation. Each stage contributes to a systematic understanding of the construction process and its mathematical justification. This approach introduces elements of mathematical inquiry and may deepen students' understanding of geometric relationships.

It is also important to identify key problems, the solution of which forms the basis for a significant number of constructive geometry problems (essentially finding individual figures and similar operations). In this context, it is appropriate to classify problems according to the geometric transformation applied in their solution. This approach helps students recognize structural similarities between problems and apply analogous solution strategies to new tasks.

Let's illustrate this in more detail using the example of parallel translation.

After introducing the concept of parallel translation, proving its properties, and solving typical problems, school students should be asked to solve a related problem (Problem 1).

Problem 1. *Construct a trapezoid by its diagonals, median and one of the angles at the base.*

For illustrative purposes, we present only the stage of analysis, which reveals the essence of the solution to the problem.

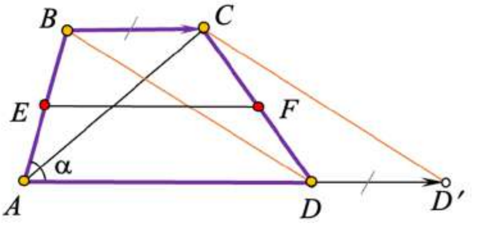


Figure 1. *Figure for solving Problem 1*

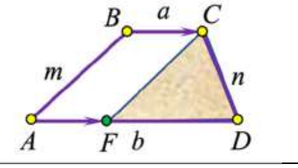
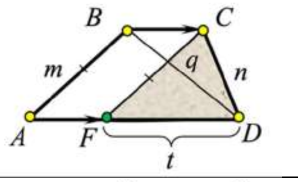
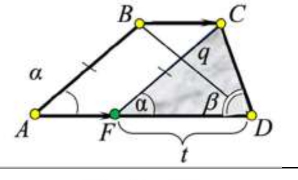
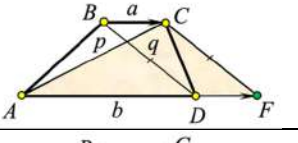
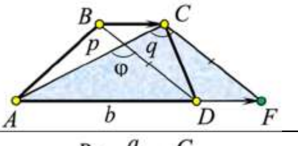
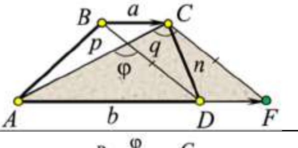
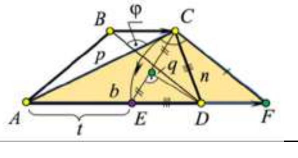
Analysis. Considering that the condition specifies both diagonals, we will try to "unite their ends". To this end, we translate the diagonal BD by the vector $\overrightarrow{BC}$: $B \to C$, $D \to D'$ (Fig. 1). We obtain that in triangle ACD' the base AD' equals twice the median of the desired trapezoid, since $AD' = AD + DD' = AD + BC = 2.EF$. Consequently, the auxiliary triangle ACD' can be easily constructed by its three sides (stage 1 of construction). Then the vertices A and C of the desired trapezoid are known. The third vertex, B, lies on the line passing through point C, parallel to AD' (stage 2). At the same time, it lies on the line AE forming a given angle α with the line AD' (stage 3). Therefore, point B is obtained as the intersection of these two lines. Since $BD \parallel CD'$, the last vertex of the trapezoid D is also easy to construct (stage 4). The analysis is complete.

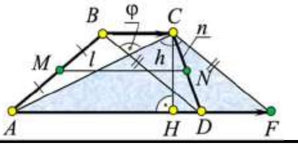
After this, the students should be asked to find similar problems. The general requirement is to solve the problem by applying translation by vector $\overrightarrow{BC}$ to one or more elements of the figure.

Consequently, through the process of searching (which requires solving a significant number of problems using the figures shown below) — with a teacher or independently — students will conclude that in such problems, it is necessary **to construct a trapezoid according to four given conditions.** At the same time, the solution will contain one key step – translation. Table 1 presents exactly such problems that students identify first. The solutions themselves are presented through figures.

To make it easier for instructors and students to use the table below, we will introduce the symbols for the elements of the trapezoid specified by the problem statement:

Table 1. Examples of problems on constructing a trapezoid whose solution uses translation by vector $\overrightarrow{BC}$

Example No.	Elements specified by the problem statement	Figure (Solution)
1	a, b, m, n	
2	$b - a = t, m, n, q$	
3	$b - a = t, q, \alpha, \beta$	
4	a, b, p, q	
5	b, p, q, φ	
6	n, p, q, φ	
7	$b - n = t, p, q, \varphi$	

8	l, h, n, φ	

- a – upper base, b – lower base;
- α, β – angles at the lower base;
- p, q – diagonals;
- φ – angle between diagonals;
- m – left side, n – right side;
- h – height;
- l – median;
- t – difference of the bases.

This approach illustrates the geometric meaning of translation while maintaining mathematical rigor.

4. The role of STEM-based design in teaching geometric transformations

Of course, the approach described above is important. However, some students, for whom the theoretical solution of a geometric problem is not an end in itself, will rightly ask about the practical application of parallel translation as well as geometric construction problems in general. In this case, it is worthwhile to involve students in the development of STEM projects. These can be focused on purely theoretical research or have practical applications. In particular, schoolchildren may be interested in robotics, creating various “manipulator arms”, developing existing models of airplanes, cars, ships, as well as fantastic or even self-invented designs. Such activities may contribute to the development of students’ design and modeling skills.

Now, we will demonstrate how, in the context of designing small radio-controlled model airplanes (or ordinary gliders), students can discover the practical significance of parallel translation.

When implementing STEM projects, students encounter several technical tasks, including the calculation of aerodynamic forces, the analysis of stability and controllability, and the determination of the aircraft's center of mass and aerodynamic center. In the authors' point of view, their search will reveal that students typically encounter one of the key concepts in aircraft design: the mean aerodynamic chord (or MAC). This is one of the most important characteristics of an aircraft wing, and its calculation is among the very first steps in the design process.

So, MAC is a reference geometric characteristic, which is the average value of the wing chord length, weighted across the entire area, and is defined as follows:

$$MAC = \frac{2}{S_W} \cdot \int_0^{\frac{b}{2}} c^2 dy \qquad (1),$$

where S_W is the wing area, c is the wing chord, and b is the wing span (Kundu, 2010, pp. 79–80). It should also be noted that the chord of a wing is a segment connecting a point on the leading edge and a point on the trailing edge of the wing in a certain section.

Usually, using integral calculus to calculate the MAC of simple models is quite inconvenient and not always advisable. For such projects, the primary task is still to teach students to apply their mathematical knowledge in practice with a minimum amount of knowledge, skills, and abilities. Such project-based work may increase students' motivation and contribute to greater confidence in applying mathematical knowledge.

Given these circumstances, it is appropriate to simplify the definition of MAC as the chord of an equivalent rectangular wing (Luk'ianov, Ed. 2024, pp. 60–61). This representation is the result of numerous experimental studies, which demonstrate that an arbitrary wing planform can be replaced in calculations with an equivalent rectangular wing of the same area.

In addition to the term MAC, several other concepts should be illustrated that will be important in further calculations. Figure 2 shows a simplified image of an aircraft wing, where c_{root} is the root chord, c_{tip} is the tip chord, and the capital letters A and B denote the corresponding left- and right-wing panels.

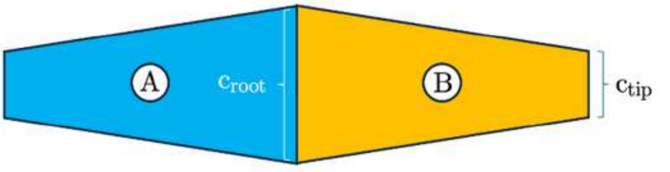


Figure 2. Schematic representation of an aircraft wing

Now let's examine a problem that is crucial to completing school STEM projects on designing and building model aircraft: *determining the mean aerodynamic chord of a wing.*

For school student projects, it is appropriate to use rectangular and swept wings in the plan. For a rectangular wing, the mean aerodynamic chord coincides with both the root and the tip chord. In contrast, for a swept wing — the panels of which resemble a trapezoid in planform — the calculation is more complex. In this case, there is a geometric method for constructing the MAC, presented in the studies (Raymer, 1992, p. 49; Luk'ianov, Ed. 2024, p. 61), and the analytical derivation of the formula for calculating its length is described in the research (Kundu, 2010, pp. 79 – 80). This results in the following expression (2)

$$MAC = \frac{2}{3}\left(\frac{\left(c_{root}+c_{tip}\right)^2}{c_{root}+c_{tip}} - \frac{c_{root}\cdot c_{tip}}{c_{root}+c_{tip}}\right) = \frac{2}{3}\cdot c_{root}\cdot\frac{1+\frac{c_{tip}}{c_{root}}+\frac{c_{tip}^2}{c_{root}^2}}{1+\frac{c_{tip}}{c_{root}}} =$$

$$= \frac{2}{3}\cdot c_{root}\cdot\frac{1+\lambda+\lambda^2}{1+\lambda} \qquad (2),$$

where $\lambda = \frac{c_{tip}}{c_{root}}$, the wing taper ratio, is another important parameter.

Using the already developed algorithm for the geometric construction of the MAC, we modify it using the parallel translation transformation.

Figure 3 shows the right-wing panel of a swept wing in planform, represented by the scalene trapezoid $ABCD$. Let us perform a parallel translation of the base AD (the root chord) along the vector $\overrightarrow{DB}$, so that $AD \to A'B$. Next, we perform another parallel translation, this time of the base BC along vector $\overrightarrow{BD}$, $BC \to DC'$.Finally, we draw segment EF joining the midpoints of the bases.

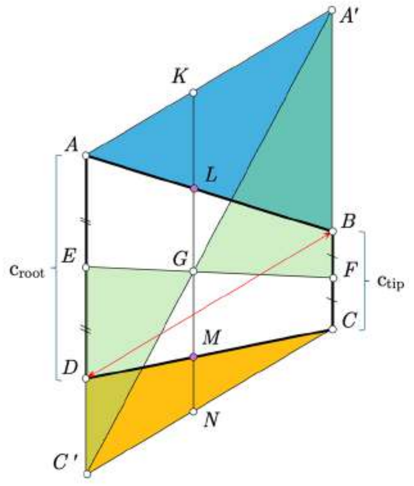


Figure 3. Geometric method of constructing MAC

The subsequent steps follow the classical algorithm: find the point $G = A'C' \cap EF$, where AC' is the diagonal of the parallelogram $AA'CC'$. Through point G, draw a line KN parallel to the bases of the trapezoid. KN intersects the sides AB and CD at the points L and M, respectively.

The segment LM will be the mean aerodynamic chord for this wing panel, and since the wing of an aircraft is symmetrical about the root chord, this chord applies to the entire wing.

The validity of this construction can be established by analyzing the similarity of the corresponding triangles. Specifically, three pairs of similar triangles should be considered: $A'FG$ and $C'EG$ (the green triangles); CMN and CDC' (the orange triangles); ALK and ABA' (the blue triangles). By determining the similarity ratio from the first pair and applying algebraic transformations, one can derive formula (2), thereby proving the construction's validity. Methodologically, it is appropriate to encourage students to derive this proof independently.

The integration of such tasks into the curriculum depends on instructional objectives and the broader educational context. Instructors can offer students purely theoretical research or, by collaborating with teachers of physics, computer science, and technology, encourage them to create a full-fledged flying model, designed using 3D modeling software and produced on a 3D printer. This is also an example of how geometric transformations can be used to demonstrate the practical application of constructive geometry. Further experimental evaluation of the proposed model for teaching parallel translation, grounded in mathematical modeling, is necessary in order to determine its methodological advantages and potential limitations within educational practice.

5. AI services in the context of learning parallel translation

Another relevant aspect in the context of digitalization is the use of artificial intelligence tools in geometry instruction. In the context of our topic, we will demonstrate how publicly available AI models and graphical mathematical tools can be utilized to aid students in learning the principles of constructive geometry and mathematics in general. To do this, based on the facts already discussed, it is worth suggesting that students practice finding the MAC for different variants of swept wings (examples with forward or backward sweep can be considered).

Artificial intelligence tools that generate images from text prompts can be used to produce schematic variants of aircraft models for instructional purposes. Such activities may help students develop skills in formulating precise prompts and promote responsible use of AI technologies. Moreover, students should be asked to create their own task based on the drawing.

The following case can be considered as an example. The ChatGPT-5 service is given the following prompt.

Example query.

You are required to produce a drawing of a single-seat aircraft. Create an image of the aircraft drawing, showing only the top view. The aircraft should have a moderately swept wing. The tip chord should be smaller than the root chord, with a taper ratio of 0.75. The drawing should be schematic. The aircraft outline must be represented with a thick continuous line. The longitudinal axis of symmetry of the aircraft must be indicated with a dash-dot line.

Figure 4. The result of a query in the ChatGPT-5 AI web service

The result of executing the query is the image shown in Figure 4.

Based on this figure, the following construction problem may be formulated, illustrating the practical relevance of concepts studied in school geometry. In our opinion, its problem statement may be as follows.

Problem 2.

Graphically plot the center of gravity (CG) of the aircraft on the generated image of the aircraft planform. The designer has indicated that the corresponding point lies on the aircraft's symmetry axis and is located at 25% of the MAC away from the leading edge of the wing.

The geometric construction is performed in three main steps: first, construct the mean aerodynamic chord (MAC) of the wing using the algorithm already discussed above; second, set aside a segment equal to one-fourth of the MAC from the leading edge along that chord; and in the final step, draw a perpendicular from the end of this segment to the aircraft's axis of symmetry. The intersection of this perpendicular with the axis will be the desired center of gravity in the drawing.

For dynamic illustration, it is appropriate to use software like GeoGebra, which allows for a clear and reasonably accurate demonstration of the construction process. Figures 5–7 illustrate the implementation of each stage, respectively.

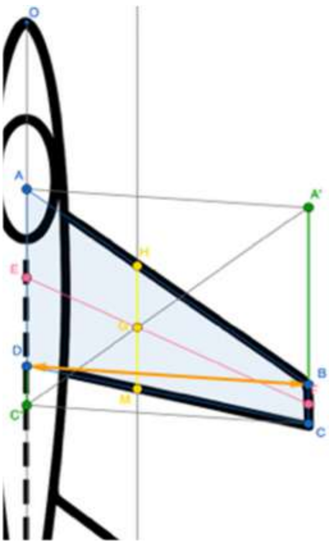


Figure 5. Step 1 of solving Problem 2

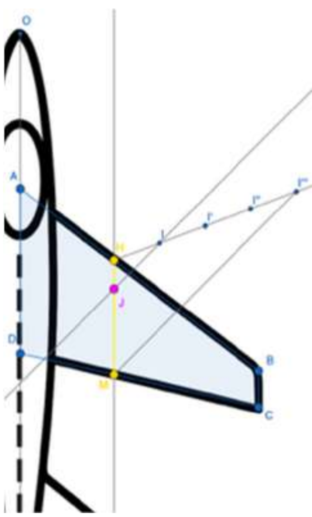


Figure 6. Step 2 of solving Problem 2

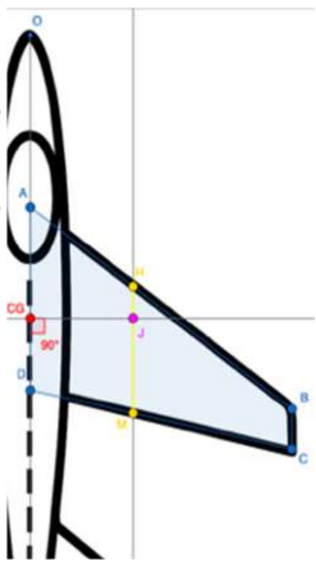


Figure 7. Step 3 of solving Problem 2

Beyond the introduction AI tools, this task may enhance students' ability to interpret graphical representations and to understand the structural role of the mean aerodynamic chord.

6. Conclusions and prospects for further research

In the authors' view, the discussion suggests that a comprehensive approach combining problem typification and STEM-based activities may improve students' understanding of geometric transformations. The integration of AI technologies may also be beneficial.

In particular, we think that the implementation of typification of construction problems as an important component of studying parallel translation can be easily extended to other planar transformations, such as rotation, homothety, line reflection, and similarity transformations. In the authors' assessment, this approach may be applied to various types of geometric constructions and illustrates potential connections between related transformations.

Within the framework of this study, STEM projects are considered a means of illustrating the practical applications of geometric transformations. At the same time, we would like to emphasize that this approach presents significant opportunities for integrating interdisciplinary connections between mathematics and other disciplines. Primary areas for integration include subjects such as physics, computer science, and technology. Brief research inquiries related to the history of aeronautical engineering will also be engaging for students. For example, the construction of scale aircraft models from different historical periods may serve as an illustrative task. It is also worth considering the possibilities of geometric transformations in the creation of geometric patterns, which can be programmed for automated generation.

It is also important to demonstrate to students the specifics of ethical use of artificial intelligence technologies. In particular, the use of such tools may expand the range of tasks available to students and support the development of critical evaluation skills.

In the authors' view, the implementation of this approach depends on the professional readiness of mathematics teachers. This entails the enhancement of their professional development.

It is essential to consider the level of student preparation when selecting tasks of suitable complexity.

In the authors' view, emphasis on the constructive component may contribute to a more comprehensive understanding of geometric transformations; however, this assumption requires practical validation.

Further research prospects should be linked to experimental verification of the vision presented in the central parts. This will ensure the detailed development of methodological materials for both the topic "Parallel translation" and other sections of the module "Geometric transformations on a plane". Such research may clarify the contribution of each component to students' mastery of geometric transformations. The claims presented in this article reflect the authors' theoretical position and require further evidence-based support.

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