MATHEMATICAL MODEL AND SOFTWARE FOR OPTIMIZING THE DISTRIBUTION OF GENERATION AMONG GAS PISTON UNITS WITHIN A SINGLE-SITE ENVIRONMENT

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Abstract. In the context of global energy transition and energy decentralization, small-scale distributed generation facilities, particularly Gas Piston Units (GPU), have become essential for ensuring the energy autonomy of industrial sites and small settlements. These units are cost-effective, compact, and could operate in co-generation cycles within local microgrids. Current energy market dynamics and national grid requirements increasingly push small businesses to deploy localized generation sites where optimization focuses on a single technological area. This paper develops a methodology for the static economic dispatch of generation among GPUs as a core component of automated control systems.
Keywords: Economic Dispatch, Distributed Generation, Gas Piston Units (GPU), Mixed-Integer Linear Programming (MILP), Static Mode Optimization.

Introduction. Global decarbonization and decentralization have increased the importance of small-scale distributed generation. Gas piston units are ideal examples, offering high maneuverability, low emissions, and cogeneration capabilities. Current energy trends are pushing small businesses to deploy these microgrids based on two concepts: autonomous "here and now" generation for a single site and rapid response to dynamic energy market requirements.

This research focuses on methods for calculating generation distribution within a small station at a single site that includes several identical GPUs. This approach abstracts from external network constraints and transmission losses between remote nodes, focusing instead on internal energy efficiency.

Power system optimization has evolved from the foundational Economic Dispatch (ED) frameworks of Wood, Wollenberg, and Sheblé (1) and the constraint management algorithms of Zhu (2) to modern studies. Recent research from 2024 and 2025, such as Wu et al. (3) and Kumar et al. (4), explores complex multi-vector systems and modern energy trading uncertainties. Today, Mixed-Integer Linear Programming (MILP) is the standard for solving these problems. As shown by Superchi et al. (5) and Villarroel et al. (6), MILP effectively models the discrete variables—such as unit startup/shutdown and step load changes—that are critical for GPU management. The issues of generation dispatch have also been extensively studied by domestic authors (7-8).

While the existing literature covers large-scale grid optimization and complex multi-source islands, there is a distinct need to investigate the Economic Dispatch problem specifically tailored for small-scale industrial sites. This practical constraint motivates the purpose of this work.

Formulation of the problem. The development of a mathematical model for optimizing the distribution of generated power among GPUs under conditions of separate generation sites, as well as to develop the software implementing this model and to test it in real-world conditions.

Solving the problem. The Economic Dispatch problem is formulated as a mathematical programming task, aimed at determining the generation levels for each of the n GPUs by minimizing the objective function while adhering to a set of technical constraints.

Definition of variables and parameters.

For each gas piston unit $i \in \{1, \dots, n\}$ the following parameters are defined:

- P_i^{min} – technical minimum gross power, MW;
- P_i^{max} – maximum rated gross power, MW;
- $ON_i(P_i^{brutto})$ – auxiliary power consumption coefficient, depending on the current load, %;
- $C_i(P_i^{brutto})$ – specific fuel costs depending on the operating mode, typically UAH/MWh.

The decision variables are:

- $u_i \subset \{0, 1\}$ – a binary variable determining the unit status (1 – on, 0 – off);

– P_i^{brutto} – the generated gross power of unit i, MW.

Since the unit's status u_i and its gross power generation P_i^{brutto} are known, we can determine the net generation by accounting for auxiliary power consumption P_i^{netto}. Furthermore, by incorporating raw material costs, we can calculate the total operating cost for the selected generation mode.

Objective function. The primary optimality criterion is the minimization of total operating fuel resource costs over the calculation time interval:

$$\min Z = \sum_i (C_i(P_i^{brutto}) + SC_i \cdot u_i), \quad (1)$$

where SC_i – startup cost for corresponding GPU.

Typically, according to the GPU documentation, its operation is not constant. It is defined by several intervals or operating modes. For each interval, we know the points (x_k, y_k), where x_k – is the gross power and, y_k – is a specific parameter of that interval. Therefore, all GPU characteristics are represented as piecewise linear functions.

Generation Characteristics. Since efficiency and auxiliary power consumption depend on the generation mode, the relationship between P_i^{netto} and P_i^{brutto} is described by a set of data points for each generation unit:

$$P_j^{netto} = f_j^{netto}(P_j^{brutto}), \quad (2)$$

where ***j*** ranges from 0 to the number of the unit's operating intervals.

Cost Characteristics. Fuel cost is assumed to be constant for a single optimization step (usually one hour). However, specific costs depend on the operating mode:

$$C_j = P_j^{brutto} \cdot b_j(P_j^{brutto}) \cdot Price_{fuel}, \quad (3)$$

where b_j represents the specific costs in interval.

Power Balance. The total net output of all units must cover the target generation, which is typically the amount of power sold on the energy market:

$$\sum_i P_i^{netto} \geq P^{target\_netto}. \quad (4)$$

Technical Generation Limits. Each unit can only operate within its specific working range.:

$$u_i \cdot P_i^{brutto} \leq P_i^{netto} \leq P_i^{max}. \quad (5)$$

Gross power constraints are also defined:

$$0 \leq P_i^{brutto} \leq u_i \cdot P_i^{\max\_brutto}. \quad (6)$$

The set of dependencies (1) – (6) forms the optimization problem. Although this problem is non-linear, it is Mixed-Integer Linear Programming (MILP), which fully describes the operation of the GPU for various generation modes.

This type of problems can be solved in different ways. In this work, we used the well-known Gurobi optimization library. In the software implementation, the procedure forms a set of rules for Gurobi and calls its solver.

Currently, one generation site includes 4-8 identical GPUs, each having the following characteristics (Table 1).

Testing of proposed method on a station consisting of 8 GPUs demonstrates satisfactory results. The search for the optimal distribution on a 12-core system is performed in approximately one second. This is completely acceptable and allows very quick prompt response to commands from the energy market operator regarding changes in generation capacity.

Table 1

Power dependable characteristics of GPU

Minimal generation, MW		1.05				
Maximal generation, MW		2.2				
Gross power, MW	Netto power, MW	Ramp up time, sec	Ramp down time, sec	Discreteness, MW	Own needs, MW	Specific cost, Cubic meter / MW
0	0	30	0	0	0	0
1.15	1.05	217	16.5	0.0023	0.1	240.66252
1.725	1.625	128	16.5	0.0023	0.1	227.080745
2.3	2.2	128	0	0.0023	0.1	222.15320

Conclusions. The article presents a mathematical model for optimizing the distribution of generation among GPUs within a single-site environment. Although it is relatively simple, it allows address several practical problems in appropriate time.

The software developed based on the proposed method is part of the automated control system for the power generation station.

Further development of the model for GPU will include accounting for transient processes during generation ramp-up and ramp-down, as well as ensuring uniform GPU loading over specified periods.

An important direction for further development requested by the business is also the extension of the model to battery energy storage systems (BESS), which includes modeling both device charging and energy discharging.

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